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Input delays, firm dynamics, and misallocation in Sub-Saharan Africa [☆]

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ABSTRACT

We quantitatively analyze the macroeconomic consequences of border delays in Sub-Saharan Africa. Delays of imported intermediate inputs lower aggregate output because of factor misallocation and due to an inefficiently low number of firms that uses foreign inputs in production. Our model economy features heterogeneous firms that endogenously differ in the degree to which foreign inputs are used. The model is calibrated to micro-level data from Sub-Saharan Africa. Reducing border delays can increase aggregate output by up to 9.4%. The gains are mainly due to a reallocation of economic activity towards more productive firms.

1. Introduction

In this paper we argue that border delays constitute a key obstacle for development. Using micro-data from Sub-Saharan Africa (SSA) we show that a large number of firms uses directly imported foreign inputs in production. These foreign inputs are often significantly delayed after arriving at a country's border. Fig. 1 shows the average time it takes from the moment goods arrive at a point of entry until the time the goods can be claimed from customs. These delays are considerable and are several orders of magnitudes larger than in richer countries, where delays are often negligible.[1]

Delays at borders can result from a variety of sources, including customs and duty procedures, sanitary regulations, a lack of modern I.T., corruption, and the fact that border posts and customs offices are often physically separated (Barka, 2012). The resulting delays occur *after* a good arrives at a point of entry and are therefore distinct from factors such as poor roads, missing infrastructure, road blocks, etc., which may delay the delivery of goods to or from a border point. To the best of our knowledge, this is the first paper to study the role of border delays in the context of a macroeconomic model with firm heterogeneity.

Border delays increase firms' costs of production for at least three reasons. First, there are opportunity costs that arise from allocating goods to storage. The second reason is depreciation, especially when production processes rely on speedy delivery. Finally, the uncertainty about delivery times creates a source of idiosyncratic risk that is specific to firms which are importing foreign inputs. Importantly, we find that such firms are larger in terms of revenue and employment. Therefore, border delays create a

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[1] Border delays across the world have increased due to the COVID-19 pandemic (see e.g. Bonadio et al. (2021)).

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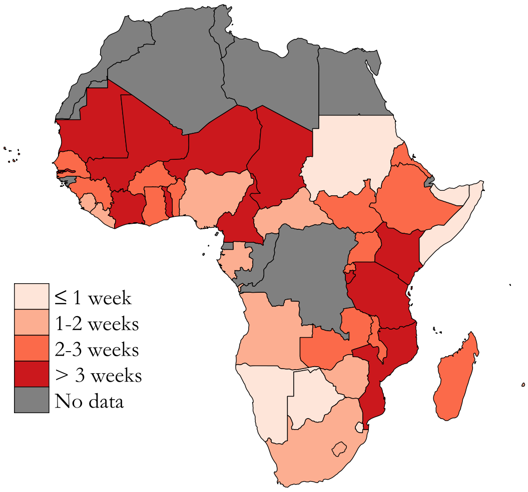

Fig. 1. Border delays in Sub-Saharan Africa. Notes.– Darker colors represent longer average delays. Delays are measured as the time it takes from the moment that goods arrive at a point of entry until the goods can be claimed from customs. Source: World Bank Enterprise Survey and authors' calculations. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

distortion that appears to be positively correlated with firm productivity (Restuccia and Rogerson, 2008). In this paper we quantify the macroeconomic consequences of these delays.

To do so, we develop a model of heterogeneous firms that differ in productivity and their production technology. Firms optimally choose whether to produce using only local inputs or to also import foreign materials. Importing foreign inputs allows firms to scale up their operation, but they must then face the risk of border delays. We allow firms to hold costly inventories to partially self-insure against that risk.

Our quantitative results imply that removing border delays would increase aggregate output by 9.4%. The positive output effects come from two channels. First, aggregate total factor productivity (TFP) increases through a better allocation of resources, away from small firms using only local inputs, towards larger, more productive firms that also use foreign inputs. Importing firms expand their scale of operation because the gains to using foreign imports increase when delays are reduced, both because of the shorter expected average delays, and because the uncertainty surrounding the delays is eliminated. Second, aggregate output increases because of a selection effect: a larger fraction of firms decides to upgrade their production technology and use foreign inputs in production. This increases output and factor demand.

Our paper contributes to the literature on the sources and consequences of factor misallocation. Persistent factor misallocation has been shown to lead to large losses in aggregate output and measured productivity (e.g. Hsieh and Klenow, 2009). Restuccia and Rogerson (2008) find that distortions which take the form of firm-specific "taxes" have the largest negative output effects when they are positively correlated with firm-level productivity. However, identifying real-world counterparts of such frictions is difficult. Often the implied output losses from directly observed distortions are small (*cf.* Restuccia and Rogerson, 2017).[2] In the context of developing countries, potential distortions include credit frictions, formality status, tax systems, and institutions.[3] We offer a new explanation, based on the prevalence of significant border delays. We find that these delays are a potential source of "correlated distortions" (Restuccia and Rogerson, 2008) because the affected firms are on average larger and likely to be more productive than firms that rely exclusively on local inputs.

Our model builds on Hopenhayn and Rogerson (1993), to which we add three sources of heterogeneity. First, firms are heterogeneous in their production technologies. We link firms' technology adoption decisions to micro-level frictions, similar to Midrigan and Xu (2014).[4] In our framework, firms face a discrete choice between using a simple production technology that uses only capital and labor, or upgrading the production technology to include first local materials, and later foreign materials. Firms with more complex production technologies operate more efficiently (Kasahara and Rodrigue, 2008; Halpern et al., 2015) by complementing the capital-labor bundle with materials in production (Oberfield and Raval, 2021; Peter and Ruane, 2022). Our second innovation is that firms which choose to produce using foreign materials expose themselves to border delays and the associated idiosyncratic delivery risk. With a given probability imported foreign materials are not available for production and any orders placed for those goods remain outstanding, possibly to be delivered at a future date. Third, we allow firms that import foreign materials to hold inventories, allowing them to self-insure against the risk of delays. Inventory management plays an important role in international

[2] Other recent examples include David and Venkateswaran (2019), Kaymak and Schott (2019), Bils et al. (2021), and Atkin and Donaldson (2021).

[3] Credit frictions are studied in Buera and Shin (2013), Midrigan and Xu (2014), Cole et al. (2016), and Itskhoki and Moll (2019). The literature on informality is summarized in La Porta and Shleifer (2014) and Ulyssea (2020). Tax systems are studied e.g. in Gordon and Li (2009). The role of institutions is summarized in Acemoglu et al. (2005).

[4] Other examples of endogenous technology adoption choices in the context of economic development are Parente and Prescott (1994) and Bustos (2011).

Table 1
Firms' characteristics by type.

	Type 1	Type 2	Type 3	All
Uses inputs of foreign origin	No	Yes	Yes	–
Direct import of foreign inputs	No	No	Yes	–
Fraction of firms (in %)	48.6	27.0	24.3	100
Fraction of employment (in %)	30.5	18.8	50.7	100
Employment	23.3	27.7	89.3	37.3
Age (in years)	14.5	15.5	18.3	15.5
Share of foreign inputs	–	–	58.0	–
Inventory of main input (in days)	–	–	31.3	–

Notes.– Moments are GDP-weighted averages across countries. Source: World Bank Enterprise Survey, the 2010 World Bank national accounts data, and authors' calculations.

trade in presence of uncertainty (Alessandria et al., 2010). The key contribution of our paper is to show how a novel type of friction, significant border delays, leads to misallocation and output losses.[5]

The delays in our model are conceptually different from standard time-to-build models (e.g. Kydland and Prescott (1982)), which typically assume a deterministic amount of time before investment projects are completed. In contrast, delays of imported materials are stochastic in our model, implying that firms cannot circumvent the friction by investing in advance, and further generating an additional negative effect through uncertainty (*cf.* Bloom (2009)).[6] Our model generates rich joint distributions of firm sizes, border delays, outstanding orders and inventories across firms with different production technologies which are broadly in line with what we find in the Sub-Saharan African micro-data.

There is an empirical literature that studies the macroeconomic effects of delays. Those papers include Hummels and Schaur (2013) and Djankov et al. (2010) who quantify the losses from each additional day a good spends in transit and time delays for exporters. Vijil et al. (2019) find that in developing countries, border delays are linked to lower survival rates for internationally operating firms. We contribute to this literature by focusing on the consequences of input delays through the lens of a quantitative heterogeneous firm model that allows for the evaluation of a range of counter-factual scenarios.

The rest of the paper is organized as follows. We present our data in section 2. Section 3 develops our heterogeneous firm model, which is used in our quantitative analysis of border delays in section 4. Section 5 concludes.

2. Data

The data used in this paper comes from the World Bank Enterprise Surveys (The World Bank (2022)). These surveys are representative of the universe of non-agricultural, formal, private sector establishments for a large number of countries, especially developing countries. The data are deemed to be of high quality and have recently been used in related papers, including Ranasinghe and Restuccia (2018), Besley and Mueller (2018), and Atkin and Donaldson (2021). Our total sample consists of 10,830 firms across 42 Sub-Saharan African countries.[7]

Importantly for our purposes, the WBES not only provides information about firm size, revenue, and inventories but also includes, *inter alia*, three questions that determine to which degree firms use foreign inputs in production and to what extend they are affected by border delays. A complete list of the countries in our sample, the variables used, and the corresponding WBES survey questions can be found in Appendix A.

As a first step, we classify firms into three different *types* based on the degree to which foreign inputs are used in production. Firms which respond that no foreign inputs were used are classified as type 1 firms. Firms that use foreign inputs, but do not import these inputs directly, are classified as type 2 firms. Firms that use and directly import a positive amount of foreign inputs are defined as type 3 firms. The latter type is potentially subject to border delays.

Table 1 shows the distribution of firms in our sample. The resulting statistics are GDP-weighted averages across all countries in our sample. The table shows that almost 50% of firms do not use any foreign inputs in production. These *type 1* firms have fewer employees and are slightly younger compared to other firms. They make up about a third of total employment. We think of these firms as small, locally operating firms without a sophisticated production technology. An example from our dataset is a sewing shop in Lagos (Nigeria) with five employees.

Type 2 firms are older and larger than type 1 firms. They make up 27% of all firms, representing slightly under 20% of employment. Type 2 firms use foreign inputs, yet do not import them directly, but purchase them in local markets. We think of these firms as medium-scale production firms that produce output with a mix of different inputs. An example from our data is a 40-employee

5 We abstract from other potential frictions, such as financial constraints (Midrigan and Xu, 2014), managerial talent (Bhattacharya et al., 2013), or other unspecified distortions as in Bento and Restuccia (2017).

6 In a related paper, Adom (2021) studies the effect of border delays in a model of ex-ante homogeneous firms. Meier (2020) studies stochastic time-to-build in U.S. business cycles due to supply chain disruptions. Carreras-Valle (2021) studies the role of increasing delivery times in the rise of inventories in the context of US-China trade.

7 The data is at the establishment level. We use the terms "establishment" and "firm" synonymously in the remainder of the paper.

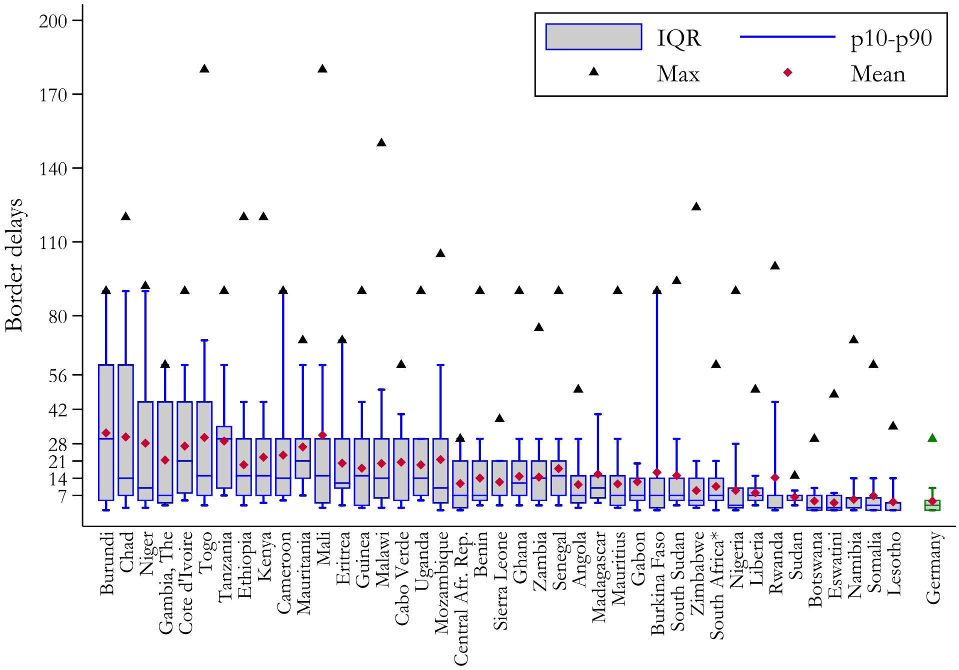

Fig. 2. Border delays in Sub-Saharan Africa. Notes.– The figure shows country-specific distributions of border delays in days. The box plots cover the 25th to 75th percentiles of delays. Medians are indicated by the horizontal bar. The lines extending from the boxes show the 10th and 90th percentiles. Averages are shown as red diamonds. The black triangles show the maximum delays. Minimum delays were zero in all cases. Source: World Bank Enterprise Survey and authors' calculations. South Africa is shown here for purposes of comparison but is excluded from the calculation of cross-country moments because of the survey year 2020/21.

firm in Nairobi (Kenya) that produces plastic bags. This firm sources all of the required inputs locally, yet 50% of them are of foreign origin. To clearly delineate firm types and to render our estimates more conservative, we assume that there is no direct effect of border delays on type 2 firms.

Finally, *type 3* firms are the largest and oldest firms on average. The 90th percentile of type 3 firm sizes is 154 employees. These firms account for 23.4% of firms and have an over-proportional employment share of over 50%. We think of these firms as relatively large, internationally operating firms that use sophisticated production technologies. An example of a type 3 firm from our data is a leather manufacturing company that employs 251 workers in Abidjan (Côte d'Ivoire). This company directly imports a fraction of its inputs and reports an average delay of 45 days for imported goods to clear the border.

To measure the importance of foreign inputs, we calculate an average share of foreign inputs in production of 58%. Firms can self-insure against delivery risk and other supply chain disruptions by holding inventories. The WBES data asks firms to report the amount of inventories of their main input. We compute that for type 3 firms inventories average slightly above one month's worth of production.

Importantly, type 3 firms import foreign inputs directly and are therefore subject to border delays. Using the pertinent survey response allows us to construct the distribution over the border delays experienced by firms. The country-specific average delays were used to construct Fig. 1. The overall GDP-weighted average delay is 13.8 days.

There is substantial heterogeneity in average border delays both across and within countries. This is shown in Fig. 2, which ranks Sub-Saharan African countries from left to right by their 75th percentile in the delay distribution.[8] Average delays range from four days in Eswatini (formerly officially entitled Swaziland) to 32.4 days in Burundi. For comparison, average delays are under five days in Germany (included on the right of Fig. 2 for reference), Sweden, and Spain, and about 6–7 days in Thailand.[9]

Fig. 2 also shows that in many countries there is a high probability to experience a long delay of over 90 days. For example, Cameroon has an average delay of 23.4 days, yet 10% of all orders experienced a delay of at least 90 days.[10]

We now develop our economic model to study the macroeconomic effects of border delays.

8 Not all countries are surveyed in the same year, but country-averages are fairly stable over time for countries where two surveys are available. For example, the average border delay in Cameroon was 23.9 days in 2009 and 23.4 days in 2016.

9 We show a distributional plot for a number of richer economies in Fig. A.1 in Appendix A.

10 These findings are confirmed by a large number of case studies. For example, Raballand et al. (2018) document high average amounts of time that cargo spends within African ports as well as the frequent occurrence of extremely long delays. United Nations (2010) estimate delays in crossing borders for products in Sub-Saharan Africa to be the lengthiest in the world. According to Portugal-Perez and Wilson (2009), the number of import procedures in Sub-Saharan Africa is among the highest in the world. The World Bank (2012) finds that an average customs transaction in Africa involves 20 to 30 different parties, 40 documents, 200 data elements (30 of which are repeated at least 30 times) and the rekeying of 60 to 70 per cent of all data at least once.

3. Model

Time is discrete and the horizon is infinite. A model period corresponds to one week.[11] The economy consists of a mass of heterogeneous firms and a representative household with discount factor $\beta \in (0,1)$. At time t a positive mass of price-taking firms produces a homogenous output good using a combination of labor, capital, and materials. The price of the output good serves as the numéraire.

While capital and labor are domestic inputs, materials can be either of domestic or foreign origin. In order to produce with foreign materials, we assume that firms must directly import them. The benefit of using foreign inputs is that they complement domestic inputs in production. The downside is that this exposes an importing firm to delivery risk, caused by delays. Firms that do not use foreign materials generate output with a simpler production function.

Production function All firms produce with a version of the following CES production function that combines capital, labor, as well as a CES composite of local and foreign materials.

$$F(k_t, n_t, m_t, x_t) = \left(\phi \left(k_t^{\alpha_1} n_t^{\alpha_2}\right)^{\rho_f} + (1-\phi) M(m_t, x_t)^{\rho_f}\right)^{1/\rho_f}. \tag{1}$$

Capital k_t and labor n_t are combined using a Cobb-Douglas production function with $\alpha_1 + \alpha_2 < 1$. Local and foreign materials, m_t and x_t, are combined into a materials bundle $M(m_t, x_t)$. In (1), ρ_f is the elasticity of substitution between the capital-labor bundle and the materials bundle, while ϕ determines the relative importance in production. Materials are combined using the following CES function:

$$M(m_t, x_t) = \left(\phi_m m_t^{\rho_m} + (1-\phi_m) x_t^{\rho_m}\right)^{1/\rho_m}, \tag{2}$$

with elasticity of substitution between local and foreign materials and their relative importance in production given by ρ_m and ϕ_m. Production of the final good is given by

$$y_t = \varepsilon_t \cdot F(k_t, n_t, m_t, x_t), \tag{3}$$

where ε_t is idiosyncratic productivity. It follows a Markov chain, $\varepsilon \in \{\varepsilon_1, \varepsilon_2, \dots, \varepsilon_K\}$, where $\Pr(\varepsilon_{t+1} = \varepsilon_k | \varepsilon_t = \varepsilon_i) \equiv \pi_{ik} \geq 0$ and $\sum_{k=1}^{K} \pi_{ik} = 1$ for each $i = 1, \dots, K$.

There exists a large mass of potential entrants, of whom a constant mass $M > 0$ enters the economy each period by paying an entry cost $c_e \geq 0$. The initial productivity level ε_0 is drawn from the stationary distribution π_0. The subsequent evolution of productivity is governed by π_{ik}. A fraction κ of incumbent firms exogenously exits the economy at the end of a period. Exiting firms cannot re-enter the market at a later period.

3.1. Firms

Following the definitions used to analyze the data, firms differ in the complexity of their production technology. We divide firms into three *types*, indexed by $j = \{1, 2, 3\}$. Firms that use the simplest production technology - using only capital and labor - are labeled type 1 firms. All firms begin operation as type 1 but can be upgraded by paying a cost. Labor is hired on a local spot market, whereas capital is accumulated. It depreciates at rate $\delta \in (0,1)$. The law of motion for capital is given by

$$k_{t+1} = (1-\delta) k_t + i_t. \tag{4}$$

Type 2 and 3 firms have a more advanced production technology that uses capital, labor, and materials in production. Materials can be of local or foreign origin. We think of local materials m as both domestically produced materials, as well as materials of foreign origin that are not imported directly by a firm. Following the data definitions, we define type 2 firms as those that only use local materials in production. Finally, type 3 firms are able to directly import foreign materials x to be used in production. Both local and foreign materials take one period time to build. In the case of foreign materials we refer to this as *placing an order* for the imported materials. As we explain in detail below, those orders can be delayed. To mitigate this risk, type 3 firms can accumulate inventories of foreign materials. Both local and foreign materials are consumed in the production process.

We now describe the three firm types in detail.

3.1.1. Type 3 firm

A type 3 firm produces using capital, labor, local materials, and foreign materials, which it imports directly. The timing is summarized in Fig. 3. At the beginning of a period, a type 3 firm has a given level of productivity ε_t, labor n_t, an existing capital stock, k_t, a stock of local materials, m_t, and an amount $\iota_{x,t}$ of foreign materials that are available to be used in production. The firm then produces using the production function in (1). Following production, the firm learns its future productivity level ε_{t+1}. It then chooses how much to invest in future capital, local, and foreign materials, and determines next period's labor.[12]

[11] Because average border delays are in the order of two weeks, choosing a model period appropriately is important to gauge the effect of delays. Similar to the labor search literature, a model period must be chosen such that the probability of success (in this case a delivery) is below unity.

[12] Because investment into capital and local materials is frictionless, assuming that the firm learns future productivity prior to investment allows us to reduce the dimensionality of the problem. Note that the upgrading decision referred to in Fig. 3 only applies to type 1 and type 2 firms.

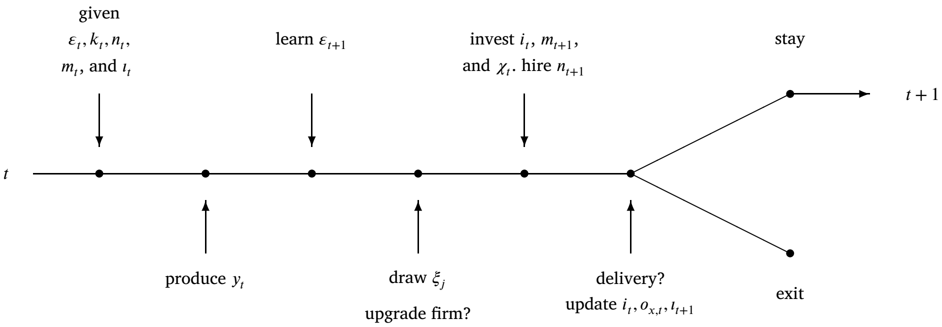

Fig. 3. Timing.

Local materials are consumed in the production process, hence the firm invests m_{t+1} units of local materials at the end of period t. At the same time, it invests an amount $\chi_t \geq 0$ into foreign materials.[13]

Import delays Foreign investments are subject to delivery risk, which we model as follows. With probability ψ all of a firm's outstanding orders $o_{x,t}$, as well as any new investment χ_t into foreign materials are delivered and available for production in the following period. In that case, future outstanding orders are equal to zero. With the counter-probability $1-\psi$, outstanding orders and new investments are *not* delivered and are not available for production.[14] The investment into foreign materials is added to the outstanding orders in that case. The law of motion of outstanding orders of foreign materials, defined after production, is therefore given by

$$o_{x,t} = \begin{cases} 0 & \text{with probability } \psi \\ o_{x,t-1} + \chi_t & \text{with probability } 1-\psi \end{cases} \tag{5}$$

In order to mitigate delivery risk, type 3 firms can hold (costly) inventories of foreign materials. To accumulate inventories, during the production phase the firm can decide the fraction of available foreign materials to spend. This fraction is denoted $\omega_t \in [0,1]$. The remaining fraction is carried over to the next period in the form of inventories. The amount of foreign materials used in production is given by

$$x_t = \omega_t \cdot \iota_{x,t}. \tag{6}$$

This is implies that the law of motion of a firm's inventory of foreign materials, defined after production, is given by

$$i_{x,t} = \begin{cases} \left(i_{x,t-1} + o_{x,t-1} + \chi_{t-1}\right) \cdot (1-\omega_t) & \text{with probability } \psi \\ i_{x,t-1} \cdot (1-\omega_t) & \text{with probability } 1-\psi \end{cases} \tag{7}$$

Finally, the available amount of foreign materials for production in the following period is given by

$$\iota_{x,t+1} = \begin{cases} i_{x,t} + o_{x,t} + \chi_t & \text{with probability } \psi \\ i_{x,t} & \text{with probability } 1-\psi \end{cases} \tag{8}$$

To fix ideas, consider the special case of no delays, i.e., $\psi = 1$. From (5) outstanding orders would always be zero and all purchased foreign materials would be available for production. There would be no need for holding inventories and $\omega_t = 1 \, \forall t$.[15]

Recursive formulation We now formulate the type 3 firm's problem in recursive form. Time subscripts are omitted in this formulation, primes indicate future values. The value function is defined right before the investment decisions are made.

The state vector of a type 3 firm is (ε, i_x, o_x), where ε denotes the idiosyncratic productivity in the following production cycle, i_x is the amount of inventories of foreign materials, and o_x is the amount of outstanding orders of foreign materials. The problem of a type 3 firm in recursive form is

$$W_3(\varepsilon, i_x, o_x) = \max_{k,n,m,\chi} \Bigg\{ -(k+m+\chi) - \theta i_x + \tag{9}$$

[13] The assumption is that re-exports of previously imported materials are ruled out (i.e., because they are prohibitively expensive) as in Alessandria et al. (2010). Previously placed, outstanding orders cannot be canceled because we assume that the delay occurs at the border of the importing country.

[14] The assumption that the probability of delivery is independent of the current delay is made for tractability.

[15] Our setup implies that in the absence of delays, foreign materials arrive one period after an order is placed. While even without border delays it may take more than one week for foreign materials to be available (Alessandria et al., 2010), we think of those "delays" as anticipated, in contrast to the uncertain delays which we model. Firms could respond to anticipated delays through accumulating inventories, which is why including anticipated delays into our model is unlikely to change our core findings.

$$\beta(1-\kappa)\left[\max_{\omega}\mathbb{E}_{\psi}\left\{\varepsilon F(k,n,m,x)-wn+(1-\delta)k+\mathbb{E}_{\varepsilon'|\varepsilon}W_3(\varepsilon',i'_x,o'_x)\right\}\right]+$$
$$\left.\beta\kappa\mathbb{E}_{\psi}\left(k+m+\iota'_x\right)\right\},$$

where $\mathbb{E}_{\varepsilon'|\varepsilon}$ denotes the conditional expectation of ε' conditional on ε and $\mathbb{E}_{\psi}$ denotes the expectation over the realization of the delivery risk. In (9) the amount of foreign materials used in production given by $x=\omega\cdot\iota'_x$, where ι'_x denotes the amount of available foreign materials in the following production cycle.[16] The choices of χ and ω are subject to $\chi\geq 0$ and $0\leq\omega\leq 1$.

The formulation in (9) implies that a firm chooses k, n, m, and invests χ at the end of the current period. It pays a resource cost of θ per unit of inventory held. If the firm does not exit (with probability κ), it enters the next period and optimally chooses a fraction ω of available foreign materials to use. It then continues into the future period with productivity ε' and the endogenous states i'_x and o'_x, which depend on the realization of the delivery risk, as well as the firms choices of investment and ω. The last line of (9) says that an exiting firm recovers its investments into capital and local materials. In addition, the firm recovers all of its available foreign materials (including inventories). Delayed orders of foreign materials cannot be recovered.

Optimal choices A firm chooses optimal capital, labor, local materials, an investment in foreign materials, as well as a fraction of available foreign materials to use in the following production cycle. The first-order conditions of (9) with respect to capital, labor, and local materials are standard and are given by

$$r=\mathbb{E}_{\psi}\varepsilon\frac{\partial F(k,n,m,x)}{\partial k}\qquad w=\mathbb{E}_{\psi}\varepsilon\frac{\partial F(k,n,m,x)}{\partial n}\qquad r_m=\mathbb{E}_{\psi}\varepsilon\frac{\partial F(k,n,m,x)}{\partial m},\tag{10}$$

where $r=\frac{1-\beta\kappa}{\beta(1-\kappa)}-(1-\delta)$ is the user-cost of capital, w denotes the wage rate, and $r_m=\frac{1-\beta\kappa}{\beta(1-\kappa)}$ is the user-cost of materials. The right-hand sides of the three equations in (10) respectively denote the expected marginal product of capital, labor, and local materials. The marginal product depends on the realization of the delivery risk: With probability ψ new and outstanding orders of foreign materials are delivered, while with the counter-probability $1-\psi$ they are not. The realization of the delivery risk also prompts a potentially different choice of ω in the next production cycle, which affects how many foreign materials x can be used in production.

The first-order condition of (9) with respect to investment in foreign materials, χ, is

$$r_x=\mathbb{E}_{\psi}\left[\varepsilon\frac{\partial F(k,n,m,x)}{\partial x}\frac{\partial x}{\partial\chi}+\mathbb{E}_{\varepsilon'|\varepsilon}\frac{\partial W_3(\varepsilon',i'_x,o'_x)}{\partial i'_x}\frac{\partial i'_x}{\partial\chi}+\mathbb{E}_{\varepsilon'|\varepsilon}\frac{\partial W_3(\varepsilon',i'_x,o'_x)}{\partial o'_x}\frac{\partial o'_x}{\partial\chi}\right].\tag{11}$$

The user-cost of foreign materials is $r_x=\frac{1-\beta\kappa\psi}{\beta(1-\kappa)}$. Due to the presence of $\psi\leq 1$, this cost is weakly larger than the cost of local materials, r_m. The right-hand side of equation (11) says that the marginal benefit of increasing investment χ consists of three parts. If the investment order is successfully delivered, the firm uses a fraction of it in current production and the remaining fraction to affect its future inventories. If the order is not delivered, the additional investment increases the stock of outstanding orders.

The first term on the right-hand side of (11) denotes the marginal effect on next period's production, which enters through a change in x. Recall that foreign materials used in production are $x=\iota'_x\cdot\omega$, where ι'_x denotes available foreign materials. From its law of motion in (8), available materials only change with χ if the order is delivered (with probability ψ). In that case ω denotes the fraction of available materials to be used in production and hence $\frac{\partial x}{\partial\chi}=\psi\omega$. The second term on the right-hand side of (11) denotes the effect of a change in χ on future inventories. Inventories change according to $\frac{\partial i'_x}{\partial\chi}=\psi(1-\omega)$, meaning that if the order is delivered inventories increase by a fraction $(1-\omega)$ for each unit increase in χ.[17] The last term in (11) denotes the effect on future outstanding orders. If a newly placed order is not delivered, future outstanding orders increase one for one with χ, i.e., $\frac{\partial o'_x}{\partial\chi}=1-\psi$.

Finally, we take the first-order conditions of (9) with respect to ω. The choice of ω is made after the realization of the delivery status and can therefore differ depending on whether or not new and outstanding orders have been delivered. The first-order condition for ω is

$$\varepsilon\frac{\partial F(k,n,m,x)}{\partial x}\frac{\partial x}{\partial\omega}+\mathbb{E}_{\varepsilon'|\varepsilon}\frac{\partial W_3(\varepsilon',i'_x,o'_x)}{\partial i'_x}\frac{\partial i'_x}{\partial\omega}=0.\tag{12}$$

The equation states that change in current production is traded-off against a change in future inventories. In particular, we have that $\frac{\partial x}{\partial\omega}=\iota'_x$ and $\frac{\partial i'_x}{\partial\omega}=-\iota'_x$, so that at the optimum $\varepsilon\frac{\partial F(k,n,m,x)}{\partial x}=\mathbb{E}_{\varepsilon'|\varepsilon}\frac{\partial W_3(\varepsilon',i'_x,o'_x)}{\partial i'_x}$.

The role of inventories Firms want to hold positive amounts of inventories in order to insure themselves against delivery risk. If an investment order fails to deliver, effective materials in (2) and hence production (1) can be increased by drawing from a firm's existing stock of inventories. An alternative to holding inventories would be to substitute foreign with local materials m. However, two factors prevent this. First, m and x are not perfect substitutes. Second, all investment choices must be made simultaneously. At the time when future local materials are chosen, the firm does not yet know whether its investment into foreign materials will

[16] The laws of motion for o_x, i_x, and ι_x are stated in (5), (7), and (8). We print them in recursive form in Appendix B.3. The same appendix also contains the complete derivations of the first-order conditions.

[17] The cost of holding inventories, θ, lowers the benefits of accumulating inventories and enters (11) through the term $\frac{\partial W_3(\varepsilon',i'_x,o'_x)}{\partial i'_x}$.

be successfully delivered. Wanting to avoid over-investing into local materials weakens the potential for buffering fluctuations in available foreign materials through this channel.[18] On the other hand, there are three reasons why firms do not hold an amount of inventories so large that it becomes effectively unconcerned by the delivery risk. These are the discount factor $\beta < 1$, the probability of exogenous exit $\kappa > 0$, and the cost of holding inventories $\theta > 0$. All three lower the expected future marginal benefit of more inventories and push the firms into using more of its available foreign materials in production today.

3.1.2. Type 2 firm

A type 2 firm produces using capital, labor, and local materials. It cannot use directly imported foreign materials. We therefore set $\phi_m = 1$ and $x = 0$. From (1) and (2), the production function is given by

$$F(k_t, n_t, m_t) = \left(\phi \left(k_t^{\alpha_1} n_t^{\alpha_2}\right)^{\rho_f} + (1-\phi) m_t^{\rho_f}\right)^{1/\rho_f}. \tag{13}$$

This also implies that $\iota_t = 0$, $i_{x,t} = 0$ and $o_{x,t} = 0$, i.e. the type 2 firm cannot invest or hold inventories of foreign materials and has no outstanding orders. The timing is the same as for a type 3 firm (*cf.* Fig. 3), with the exception that after the firm learns its future productivity level, it draws a random cost of upgrading, $\xi_{2,t}$ which, if paid, lets the firm upgrade to a type 3 firm. We interpret the fixed cost ξ_2 as a technology adoption cost which represents fundamental changes in a firm's scope, customer base, and production technology.

Recursive formulation The value function $W_2(\cdot)$ is defined after production and after future productivity is learned, but before the upgrading decision has been made. The state vector of a type 2 firm is (ε). Every period, firms draw a realization of ξ_2 from a time-invariant uniform distribution $G_2(\xi_2)$, common across plants, as in Thomas (2002). The lower bound of the distribution is zero and the upper bound is denoted Ξ_2. The stochastic nature of the fixed cost gives rise to a threshold rule for $\bar{\xi}_2$. If the realized draw of ξ_2 is below the threshold value $\bar{\xi}_2$, the firm will upgrade, otherwise it will continue as a type 2 firm. The firm value is therefore given by

$$W_2(\varepsilon) = \max_{\bar{\xi}_2} \int_{\bar{\xi}_2}^{\Xi_2} V_2(\varepsilon) dG(\xi_2) + \int_0^{\bar{\xi}_2} \left[-\xi_2 + W_3(\varepsilon, 0, 0) \right] dG(\xi_2). \tag{14}$$

If the cost is paid, a firm is immediately upgraded to a type 3 firm and also invests into foreign materials, which may be available for production next period, depending on the realization of the delivery risk. A new type 3 firm starts without inventories and without outstanding orders of foreign materials. If the fixed cost is not paid, i.e., when $\xi_2 > \bar{\xi}_2$, the firm will remain of type 2 with associated value

$$V_2(\varepsilon) = \max_{k,n,m} \left\{ -(k+m) + \beta\kappa(k+m)\beta(1-\kappa)\left[\varepsilon F(k,n,m) - wn + (1-\delta)k + \mathbb{E}_{\varepsilon'|\varepsilon} W_2(\varepsilon')\right] \right\}. \tag{15}$$

From (15), the firm chooses future capital, labor, and materials at the end of the current period. Next period, if the firm does not exit, it produces using k, n, and m. It draws a new idiosyncratic productivity level and can make an upgrading choice at the end of the period, captured by $W_2(\varepsilon')$. The last term of (15) says that if the firm exits, it recovers the value of its capital stock and local materials.

Optimal choices The solution to (14) are four policy functions for k, n, m, and $\bar{\xi}_2$. The first-order conditions of (15) with respect to capital, labor, and local materials are given by

$$r = \varepsilon \frac{\partial F(k,n,m)}{\partial k} \qquad w = \varepsilon \frac{\partial F(k,n,m)}{\partial n} \qquad r_m = \varepsilon \frac{\partial F(k,n,m)}{\partial m}. \tag{16}$$

The user-cost of capital r and of local materials r_m were defined above. The firm chooses its three factors of production so as to equalize the marginal cost with the marginal benefit in production.

Finally, we derive the optimal cutoff value $\bar{\xi}_2$. From (14), the cutoff value $\bar{\xi}_2$ which makes firms indifferent between upgrading or not, has a closed form solution which does not depend on the particular distribution of the fixed cost. The cutoff, if interior, is given by the difference between the value of upgrading the firm or not:

$$\bar{\xi}_2(\varepsilon) = \min\{\Xi_2, \max\{0, W_3(\varepsilon, 0, 0) - V_2(\varepsilon)\}\} \tag{17}$$

Additional results and closed-form solutions for the firm's labor and local materials policy functions can be found in Appendix B.2.

3.1.3. Type 1 firm

A type 1 firm produces using only capital and labor. It does not use local or foreign materials, i.e. $\phi = 1$. From (1) the production function is given by

[18] Making local materials storable as well would strengthen this channel. However, the same costs of holding inventories discussed in this paragraph still apply.

$$F(k_t, n_t) = k_t^{\alpha_1} n_t^{\alpha_2}. \tag{18}$$

After the firm learns its future productivity level, it draws a random cost of upgrading, $\xi_{1,t}$ which, if paid, lets the firm upgrade to a type 2 firm.

Recursive formulation The value function $W_1(\cdot)$ is defined after production and after future productivity is learned, but before the upgrading decision has been made. The state vector of a type 1 firm is (ε). Every period, firms draw a realization of ξ_1 from a time-invariant uniform distribution $G_1(\xi_1)$ with lower bound zero and upper bound Ξ_1. The firm value is given by

$$W_1(\varepsilon) = \max_{\bar{\xi}_1} \int_{\bar{\xi}_1}^{\Xi_1} V_1(\varepsilon) dG(\xi_1) + \int_0^{\bar{\xi}_1} \left[-\xi_1 + V_2(\varepsilon) \right] dG(\xi_1). \tag{19}$$

If the cost ξ_1 is paid, a firm is upgraded to a type 2 firm and can also choose local materials for the next production cycle. If the fixed cost is not paid the firm will remain of type 1 with associated value

$$V_1(\varepsilon) = \max_{k,n} -k + \beta(1-\kappa) \left[F(k,n) - wn + (1-\delta)k + \mathbb{E}_{\varepsilon'|\varepsilon} V_1(\varepsilon') \right] + \beta \kappa k. \tag{20}$$

From (20), the firm chooses future capital and labor at the end of the current period. Next period, if the firm does not exit, it produces using k and n. It draws a new idiosyncratic productivity level and can make an upgrading choice at the end of the period, captured by $W_1(\varepsilon')$. The last term in (20) says that if the firm exits, it recovers the value of its capital stock.

Optimal choices The solution to (19) are three policy functions for k, n, and $\bar{\xi}_1$. The first-order conditions for capital and labor are

$$r = \varepsilon \frac{\partial F(k,n)}{\partial k} \qquad w = \varepsilon \frac{\partial F(k,n)}{\partial n}, \tag{21}$$

while the optimal cutoff value $\bar{\xi}_1$ is given by

$$\bar{\xi}_1(\varepsilon) = \min\{\Xi_1, \max\{0, V_2(\varepsilon) - V_1(\varepsilon)\}\}. \tag{22}$$

Additional results and closed-form solutions for the firm's policy and value functions can be found in Appendix B.1.

3.2. Entrant's problem

New entrants start as type 1 firms. A potential entrant begins operating if the value of entry exceeds the entry cost: $V^e \geq c_e$. The entry costs reflect the cost of identifying a business idea, a target market, as well as all costs associated to setting up a business. The value of entry is the expected value of a type 1 firm.

$$V_e = \sum_{k=1}^{K} \pi_0(\varepsilon_k) V_1(\varepsilon_k) \tag{23}$$

This formulation implies that upon paying c_e, entrants' initial productivity level is revealed. A new entrant invests into capital and local materials and begins operating in the following period.

3.3. Distribution of firms

Let $\mathbf{\Gamma}_j(s_j)$ denote distribution of incumbent type j firms over the relevant firm states s_j for $j = \{1,2,3\}$. With $\mathcal{M}$ denoting the mass of entrants, the law of motion for the distribution of incumbent type 1 firms is given by

$$\mathbf{\Gamma}_1{}'(\varepsilon_k) = \sum_{i=1}^{K} (1-\kappa) \left(1 - G_1(\bar{\xi}_1(\varepsilon_i))\right) \pi_{ik} \mathbf{\Gamma}_1(\varepsilon_i) + \pi_0(\varepsilon_k)\mathcal{M}. \tag{24}$$

The first term represents the evolution of current type 1 firms that do not exit and do not upgrade. The second term represents new entrants.

The law of motion for incumbent type 2 firms is

$$\mathbf{\Gamma}_2{}'(\varepsilon_k) = \sum_{i=1}^{K} (1-\kappa) \left(1 - G_2(\bar{\xi}_2(\varepsilon_i))\right) \pi_{ik} \mathbf{\Gamma}_2(\varepsilon_i) + \sum_{i=1}^{K} G_1(\bar{\xi}_1(\varepsilon_i)) \pi_{ik} \mathbf{\Gamma}_1(\varepsilon_i). \tag{25}$$

The first term represents current type 2 firms that do not exit and do not upgrade. The second term represents current type 1 firms that upgrade to type 2 firms.

Finally, the law of motion for incumbent type 3 firms is

$$\mathbf{\Gamma}_3{}'(\varepsilon_k, i'_x, o'_x) = (1-\kappa) \sum_{i=1}^{K} \mathcal{I}(\varepsilon_i, i_x, o_x) \pi_{ik} \mathbf{\Gamma}_3(\varepsilon_i, i_x, o_x) + \sum_{i=1}^{K} G_2(\bar{\xi}_2(\varepsilon_i)) \pi_{ik} \mathbf{\Gamma}_2(\varepsilon_i). \tag{26}$$

The first term represents the current type 3 firms that do not exit. The indicator function $\mathcal{I}(\varepsilon_i, i_x, o_x)$ takes the value ψ if the values of i'_x and o'_x which are being updated are implied by a successful delivery and the policy functions in firm state $(\varepsilon_i, i_x, o_x)$. The indicator function takes the value $1-\psi$ if the order is not delivered and the policy functions imply the same future values i'_x and o'_x. Using a law of large numbers, we assume that orders for a fraction $1-\psi$ of firms are delayed. The second term in (26) represents current type 2 firms that upgrade to type 3 firms.

In the stationary equilibrium the distributions $\boldsymbol{\Gamma}_j(s_j)$ for $j = \{1, 2, 3\}$ are constant over time. With $\kappa > 0$ the mappings defined above have unique invariant distributions associated with them. Additionally, the distributions are linear in the mass of entrants $\mathcal{M}$.

3.4. Households

A representative household derives utility from consumption and leisure, with $U(C,N) = \log(C) - \frac{\lambda}{1+\frac{1}{\zeta}} \cdot N^{1+\frac{1}{\zeta}}$. The term λ determines the disutility from hours worked, while ζ is the inverse Frisch elasticity. Consumption is given by total disposable income, which consists of labor earnings plus dividends: $C^d = D + w \cdot N^s$. Dividends D are the aggregate profits that are distributed from firms to households. The investment decision is handled by firms.[19]

3.5. Competitive equilibrium

A stationary recursive competitive equilibrium consists of type-specific value functions $V_1(\varepsilon)$, $V_2(\varepsilon)$, $V_3(\varepsilon, i_x, o_x)$, a value of entry V_e, type-specific policy functions for labor, capital, local materials, and investment into foreign materials, cutoff values for upgrading costs $\bar{\xi}_1$ and $\bar{\xi}_2$, a wage rate w, distributions $\boldsymbol{\Gamma}_j$ for $j = \{1, 2, 3\}$, and a measure of entrants $\mathcal{M}$ such that:

1. For $j = \{1, 2, 3\}$, the value function V_j together with the associated policy functions and cutoffs solves the respective incumbent firm's problem.
2. The free entry condition holds: $V_e = c_e$.
3. The type-specific distributions over incumbent firms are stationary: $\boldsymbol{\Gamma}'_j = \boldsymbol{\Gamma}_j$ for $j = \{1, 2, 3\}$.
4. Markets clear:
$\sum_{j=1}^{3} \int n^j(s_j) d\boldsymbol{\Gamma}_j \cdot \mathcal{M} = N^s = N^*$ and $C = Y - I^k - I^m - I^x - I^{\xi} - \mathcal{M} \cdot c_e$,

The last equation gives the resource constraint of the economy. Total output is $Y = \sum_{j=1}^{3} \int y(s_j) d\boldsymbol{\Gamma}^j \mathcal{M}$. The term I^k denotes aggregate capital investment by all firms, net of non-depreciated capital and the scrap value of existing firms. Similarly, I^m and I^x denote aggregate investment into local materials and foreign materials. Resources used for upgrading firms are $I^{\xi} = \frac{1}{2}\left[\frac{\bar{\xi}_1(\varepsilon)^2}{\Xi_1}\boldsymbol{\Gamma}^1 + \frac{\bar{\xi}_2(\varepsilon)^2}{\Xi_2}\boldsymbol{\Gamma}^2\right] \cdot \mathcal{M}$. The term $\mathcal{M} \cdot c_e$ denotes the total resources spent on firm entry.[20]

4. Quantitative analysis of border delays

In this section, we use a quantitative version of the model developed in the previous section in order to assess the effects of border delays in Sub-Saharan African countries. We calibrate the model to match the firm size and employment distributions from African data, targeting average border delays of about 14 days. In our main counterfactual scenario we compare this economy to an alternative one in which border delays are eliminated. This allows us to quantify the impact of border delays on macroeconomic outcomes.

4.1. Computational strategy

To solve our model numerically, we use value function iteration with interpolation. The equilibrium wage is found via bisection. To reduce the number of necessary state variables, we exploit the fact that capital and materials can be adjusted without frictions and that firms learn future productivity before investment decisions are made.[21] In the quantitative part, we allow for type-specific means of ε in order to match the data on employment sizes.

For type 3 firms we solve for the optimal choices of k and m by jointly solving the first-order conditions for capital and local materials in (10) numerically for every combination of ε, i_x, o_x as well as every possible choice of χ and ω (conditional on the delivery status), i.e. an eight-dimensional problem. We then proceed to solving the policy functions for χ and ω. The choice of ω depends on whether or not a firm's orders of foreign materials have been delivered or not. The joint choice of χ and ω then implies optimal choices of k and m for every state (ε, i_x, o_x). The problem of type 2 firms is solved by using a numerical solver on the firm's first-order conditions. Value function iteration is used to determine the value of not upgrading the firm, which allows us to find the

[19] We study the robustness of our results when labor supply is exogenous in Appendix C. Our main conclusions are not affected by the assumptions on labor supply.

[20] These equations imply aggregate dividends are $D = Y - w \cdot N^* - I^k - I^m - I^x - I^{\xi} - \mathcal{M} \cdot c_e$.

[21] By adding non-depreciated capital to current profits, we effectively assume that the entire stock of capital is repurchased every period. Introducing investment risk is unlikely to have a big quantitative impact on our results. This is because the model period is one week, implying a high persistence of idiosyncratic productivity shocks. We choose not to model investment risk because it would no longer be possible to solve for the optimal labor policy of type 3 firms analytically.

Table 2
Externally calibrated parameters.

Parameter	Value	Meaning
β	0.95	Discount factor (annual)
α_1	0.28	Capital income share
α_2	0.57	Labor income share
δ	0.10	Depreciation rate (annual)
κ	0.10	Exogenous exit rate (annual)
λ	0.86	Household disutility of labor
ζ	0.79	Inverse Frisch elasticity
ψ	0.41	Average import delay of 13.8 days
ϕ_m	0.40	Local materials share
ϕ	0.69	Capital-labor bundle share
ρ_f	-2/3	Substitutability of capital-labor and materials
ρ_m	1/2	Substitutability of materials

threshold value of $\bar{\xi}_2$. For type 1 firms, the policy functions for capital and labor, as well as the value function of not upgrading the firm are analytical objects, while the threshold value $\bar{\xi}_1$ is found with value function iteration.

4.2. Calibration

A number of parameters are preset, while others are calibrated to match data moments. The parameters set outside the model are shown in Table 2. Their values are conventional and are taken from the existing literature. We target an annual real rate of 5%, implying $\beta = 0.95$ at the annual frequency. We set α_1 and α_2 according to their income shares of $2/3$ and $1/3$, assuming a span-of-control parameter of 0.85 as in Restuccia and Rogerson (2008). The annual depreciation rate is set to 10%, as is the annual exit rate κ. We set the inverse Frisch elasticity to 1.2, which is in the range suggested by Chetty et al. (2011). The value of λ is set to generate a labor supply of 1 in the baseline economy.

A key parameter is the average delay of foreign materials. We compute an average delay for imported foreign inputs of 13.8 days in the WBES data. This implies a weekly delivery probability of $\psi = 0.41$.[22] From the WBES data we compute an average share of materials that are of foreign origin, which is close to 60%. This implies $\phi_m = 0.40$ in the CES for materials, (2). To determine ϕ, the importance of the capital-labor bundle in production, we use World KLEMS data to compute a materials cost share of 31%, implying $\phi = 0.69$.[23] Finally, we set the substitutability parameters $\rho = -\frac{2}{3}$ and $\rho_m = \frac{1}{2}$. This implies that local and foreign materials are substitutes in production, with an elasticity of substitution of 2, while the capital-labor bundle and the materials bundle are complements, with an elasticity of substitution of 0.6 (Peter and Ruane, 2022; Oberfield and Raval, 2021).

The remaining parameters are $\Theta = \{\mu_\varepsilon^1, \mu_\varepsilon^2, \mu_\varepsilon^3, \rho_\varepsilon, \sigma_\varepsilon, \Xi_1, \Xi_2, c_e, \theta\}$. The first three parameters denote type-specific productivity shifters, ρ_ε and σ_ε refer to the annual serial correlation and standard deviation of the productivity shock process. Ξ_1 and Ξ_2 are the upper bounds of the upgrading cost distribution and θ denotes the annual cost of holding inventories. The vector Θ is found by matching data moments that are informative about the joint distributions of firms, employment, and firm types, as well as inventories. The parameters were chosen to minimize the average percentage difference between data and model moments. The calibrated parameter values are shown in Table 3, together with a comparison between data and model moments. The parameters are jointly identified, yet we briefly describe the intuition behind the identification and our choice of data moments.

The idiosyncratic productivity shifters $\mu_\varepsilon^1, \mu_\varepsilon^2, \mu_\varepsilon^3$ identify the average firm sizes, while ρ_ε, and σ_ε determine employment concentration in large firms. As the data is organized in employment bins we target the fraction of firms in the largest size bin (100+ employees), as well their employment share. The upper bounds of the distributions of upgrading costs determine the distribution of firms across types. We therefore identify these parameters from the fraction of type 1 and type 2 firms in the data. From the free-entry condition (23) the entry cost c_e is key in pinning down the wage rate. Because the wage bill represents a larger share of the total factor costs for type 1 firms, higher wages imply a lower fraction of total employment in those firms. Finally, we pin down θ, the cost of holding inventory, by targeting the average days of inventories held by firms, which we obtain from the WBES.

Table 3 shows that the model matches the data moments well. A key data feature that is replicated by the calibrated model is that most firms are small, while employment is concentrated among large firms. This is also true within firm-types. In particular, the model generates both small and large type 1 and type 2 firms due to the existence of the variance in productivity shocks and the stochastic upgrading costs. The fraction of small type 3 firms and their share in employment is roughly in line with the WBES data.

[22] This assumes that there is no time lag between investment and delivery to the importing country's border. Alessandria et al. (2010) find that lags between orders and delivery in international trade are 1–2 months. Those additional delays could be incorporated into our analysis, but here we chose to focus on the output losses from the less predictable border delays.

[23] Using the WBES data on type 2 and type 3 firms we find the GDP-weighted materials cost share to be 0.33.

Table 3
Internally calibrated parameters.

Parameter	Value	Moment	Data	Model
μ_ε^1	−2.50	Average employment of type 1 firms	23.27	23.59
μ_ε^2	−0.64	Average employment of type 2 firms	27.69	27.95
μ_ε^3	−0.15	Average employment of type 3 firms	89.29	83.67
ρ_ε	0.72	Share of large firms (100+)	6.66	6.45
σ_ε	0.05	Employment share of large firms (100+)	47.05	44.25
Ξ_1	9.54	Fraction of type 1 firms	48.64	48.85
Ξ_2	11.40	Fraction of type 2 firms	27.04	25.28
c_e	147.12	Employment share of type 1 firms	30.52	28.64
θ	0.03	Average days worth of inventory held	31.31	31.23

Table 4
Consequences of elimination of border delays.

Average border delays (in days)	Benchmark	No delays
	13.8	0
Aggregate output	1.00	1.09
Labor	1.00	0.99
Capital	1.00	1.02
Local materials	1.00	1.04
Foreign materials	1.00	1.27
Outstanding orders	1.00	0
Inventories	1.00	0
Wage rate	1.00	1.03
Mass of firms	1.00	0.90
Fraction of type 1 firms	0.49	0.40
Fraction of type 2 firms	0.25	0.28
Fraction of type 3 firms	0.26	0.32
Employment share type 1 firms	0.29	0.18
Employment share type 2 firms	0.18	0.14
Employment share type 3 firms	0.54	0.68

Note: The values in the middle block have been normalized to one in the baseline economy.

For example, the model generates a fraction of 73.9% of type 3 firms that have less than 50 employees, making up 16.7% of type-3 firms' employment. In the data, the same figures are 71% and 19.3%.[24][25]

Overall, the model captures salient features of Sub-Saharan African economies. This implies that the quantitative model is a useful framework to study the consequences of a reduction in border delays for macroeconomic aggregates, to which we turn to next.

4.3. The effects of border delays

We now simulate a counterfactual economy in which border delays are eliminated, implying $\psi = 1$. A key finding is that reducing border delays from their current levels down to zero results in an increase of aggregate output of 9.35%.

The full results are presented in Table 4. The column 'Benchmark' shows results from the calibrated economy described above. The second column, 'No delays' reports moments from an otherwise identical economy in which $\psi = 1$ and where the wage and mass of firms have been adjusted to ensure that the free-entry condition continues to hold and markets clear.[26]

In the economy without delays, firms no longer face any delivery risk. This increases the expected return to investing in foreign materials. Consequently, aggregate foreign materials used in production increase significantly, while undelivered orders and inventories fall to zero. The elimination of investment risk also leads to positive effects on investment in capital and local materials. Firms no longer face the probability that foreign material input delays imply an *ex-post* suboptimal ratio of labor, capital, and materials to the amount of foreign materials used in production, which increases their expected marginal product. The fact that labor demand decreases relative to the benchmark is explained by the increase in the wage rate, as well as the income effect on labor supply.[27]

Because the elimination of delivery risk increases the value for all firms, the wage rate must increase for the free-entry condition to hold. Similarly, because output increases at all firms, a lower equilibrium mass of firms is required to clear the labor market.

[24] The WBES data coverage is generally better for manufacturing than non-manufacturing firms. For a subsample of countries where non-manufacturing data is available we can show that input delays are prevalent for both manufacturing and non-manufacturing firms. See Table A.4 in the Appendix for details.

[25] The WBES is meant to be representative of firms with at least five employees. Our baseline sample includes all firms, of which about 7% have less than five employees. The last column of Table A.4 in the Appendix shows that removing those firms from the sample results in almost identical target moments.

[26] We derive the corresponding model without frictions in Appendix B.4.

[27] In Appendix C we show that with fixed labor demand, our results remain very similar. The overall output gains are 10.7% in that case.

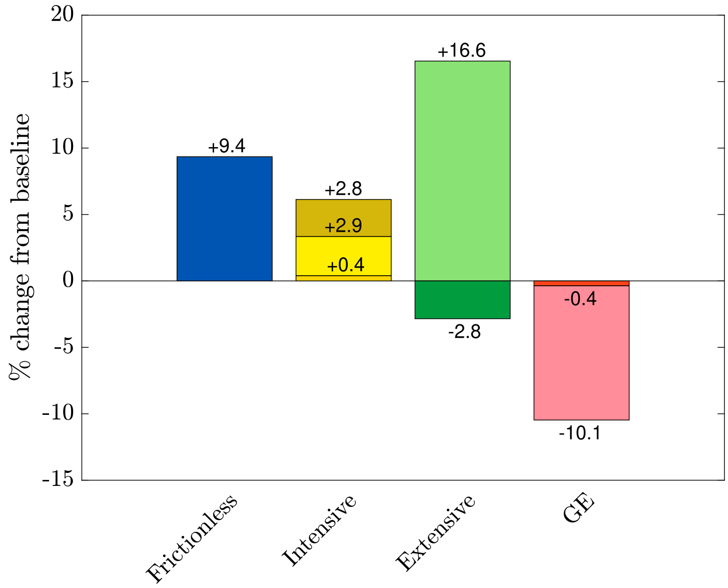

Fig. 4. Decomposition. Notes.– Bars represent output gains relative to the baseline economy with import delays. From top to bottom the second bar shows the effects of *i)* more factor accumulation, *ii)* the gap in output based on delivery status, and *iii)* the TFP gain from optimally allocating existing resources among type 3 firms. The third bar shows the change in output from *i)* changes in the firm type distribution and *ii)* the productivity distributions across types. The last bar shows general equilibrium (GE) effects coming from a higher wage rate and a lower equilibrium mass of firms.

The bottom half of Table 4 shows the effects on the distribution of firms. The key result is that the fraction of type 2 and type 3 firms increases, at the expense of type 1 firms. Because type 3 firms use foreign materials in production, the removal of border delays directly benefits these firms, increasing their value. This in turn increases the threshold value $\bar{\xi}_2$, which pins down the maximum fixed cost that type 2 firms are willing to incur to upgrade. This then increases the value of type 2 firms and leads to more type 1 firms upgrading as well. In this way, the removal of border delays creates an extensive-margin effect that leads to an increase in the fraction of firms that use foreign materials in production by over six percentage points. In terms of employment the effects are even stronger. Because type 3 firms now accumulate more capital, local, and foreign materials than in the economy with border delays, they also have a higher demand for labor. The employment share of type 3 firms increases from 54% to 68%. This shows that border delays lead to significant distortions, as the economy is inefficiently allocating resources towards less productive firm types.[28]

Decomposition Aggregate output increases by 9.4% when border delays are eliminated. We decompose these gains based on the following formulation of aggregate production:

$$Y = M^* \cdot \left(\sum_{j=1}^{3} \sum_{i=1}^{K} y_{ij} f_{i|j} g_j \right) \tag{27}$$

Output is the product of the mass of firms, M^* and the sum of output by firms in all productivity states i and all three types j, weighted by the conditional probability distributions of productivity, $f_{i|j}$, and types, g_j. Based on this, the total change in aggregate output can be decomposed into an intensive margin, an extensive margin, and general equilibrium effects.

The results of the decomposition are shown graphically in Fig. 4. The first bar represents the 9.4% increase in output when import delays are removed, corresponding to a frictionless version of our model.

The second bar represents the intensive margin changes. By intensive margin we mean the effects that come from changes in firm behavior, leaving the composition and total mass of firms unchanged (i.e., from changes in the y_{ij} terms in (27)). There are three effects. First, the top portion of the second bar represents the change in output that is the result of increased factor demand. By removing the import delay friction, firms have an incentive to accumulate more capital and invest more into local and foreign materials, as was shown in Table 4. This channel increases output by 2.8%. The middle portion represents a 2.9% increase in output which comes from eliminating the ex-post differences in firm behavior based on whether or not their orders have been delivered. If the output of firms whose orders were not delivered was not distorted (because resources are allocated to outstanding orders instead of available inputs), output would increase by 2.9%. Finally, the bottom portion of the second bar, is the change in measured total factor productivity (TFP). It amounts to an increase in output of 0.4% and was computed as the output gain that could be achieved

[28] We show results of an alternative calibration in which we remove firms with less than five employees from the data- and model-generated moments in Appendix C. The fit of that model is not as good as the model used in the analysis described above and we find output gains of 6.4% from removing import delays.

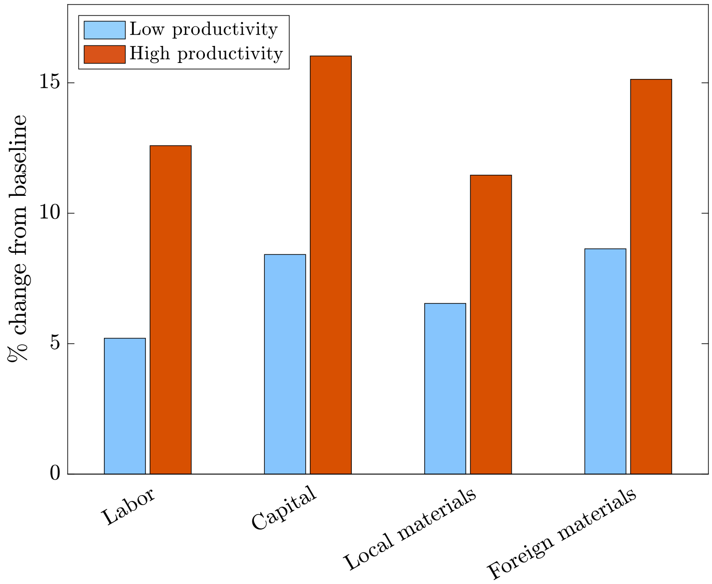

Fig. 5. Change in factor demand by productivity for type 3 firms. Notes.– The bars represent the percentage change in factor demand of type 3 firms when going from the baseline economy with frictions to the frictionless economy without delays. The light blue bars show changes for low productivity firms, the dark red bars show changes for high productivity firms. The mass of type 3 firms and their distribution across idiosyncratic productivity states is held constant.

by a social planner that uses the exact same inputs that are available in the economy with frictions and that keeps the firm type distribution constant.[29]

The third bar represents the extensive margin changes in output. By extensive margin we mean changes in the composition of firms, holding everything else constant. We compute two effects by holding all terms in (27) constant, except respectively the $f_{i|j}$ and the g_j terms, which we insert from the economy without delays. The top portion of the third bar shows that the endogenous selection into firm types is an important driver of the aggregate output effects. Without frictions, more firms choose to pay the fixed upgrading costs to eventually become type 3 firms, which are larger and more productive on average. If the firm type distribution g_j were the same as that in the economy without frictions, output would increase by 16.6%. On the other hand, the distribution of productivity across firm types changes in a way that reduces output when going to the frictionless economy. The reason for this is that as the threshold values for upgrading increase, more firms upgrade. This lowers the average productivity of type 3 firms (and increases the average productivity of type 1 firms). In a comparative static sense, this leads to a fall in output of 2.8%.

Finally, there are two general equilibrium effects, *i)* an increase in the wage rate and *ii)* a reduction in the mass of firms (*cf.* Table 4). To compute the effect of the higher wage rate, we solve the baseline economy, while imposing the wage rate that results from the frictionless economy. This partial equilibrium exercise shows that this channel reduces output by 0.4%. Table 4 reports a ten percent decline in the number of firms, which enters aggregate output linearly as per (27). We compute the partial effect on output from this channel by inserting the equilibrium mass of firms from the frictionless economy into (27), while holding all else at baseline levels. The light, bottom portion of the last bar in Fig. 4 shows a drop in output from this channel of about ten percent.

Delays as correlated distortions Border delays lead to a misallocation of factors. More specifically, they create a distortion which is positively correlated with productivity. The literature on factor misallocation has identified such “correlated distortions” as being able to generate real effects of considerable magnitude (*cf.* Restuccia and Rogerson, 2008). In the context of our model, there are two reasons for a positive correlation of the distortion with firm-level productivity. First, only firms that import foreign inputs (type 3 firms) are directly affected by the friction. In addition, the fixed cost of upgrading is more likely to be paid by firms with higher idiosyncratic productivity. This implies that in equilibrium, type 3 firms are on average more productive. The import delay friction is therefore strongest for firms with higher average productivity.

Second, the severity of the distortion is also positively correlated with firm productivity within firm types. Fig. 5 shows the changes in average factor demand of type 3 firms when delays are removed. While all firms demand more labor, capital, local materials, and foreign materials once the friction is removed, this increase is larger for high productivity firms (shown as the dark red bars in Fig. 5).

Finally, the distortion also interacts with *changes* in firm-level productivity. Type 3 firms which receive a negative productivity shock and wish to shrink, can run down their existing inventories of foreign inputs. On the other hand, firms with positive productivity

[29] Only output among type 3 firms is distorted relative to the first-best. Output of type 3 firms would increase by 0.6% if factors were optimally allocated across firms, leading to an increase in aggregate output of 0.4%. This misallocation exists also among firms whose orders were delivered because the presence of inventories and outstanding orders affects firm’s optimal behavior.

shocks that want to accumulate inventories of foreign inputs will be subject to the delay friction. The impact of the friction is therefore higher when the gap between current and target factor demand is larger.

Border delays as a tax The misallocation literature often represents distortions in the form of wedges, or 'taxes' on factor demand (*cf.* Restuccia and Rogerson (2008), Hsieh and Klenow (2009)). In the model described above, we explicitly modeled a concrete distortion and found it to be quantitatively important. An alternative approach would be to completely abstract from the exact type of friction and simply model border delays as a 'tax'. In our model, border delays increase the cost of foreign materials by creating uncertainty about the time of their delivery. We therefore ask: What would be the tax on investments of foreign materials that generates the same aggregate output effects as the benchmark model with delays?

To answer this question, we start from our baseline model but eliminate all border delays (i.e. $\psi = 1$). We then introduce a tax τ on investments in foreign materials, which were labeled χ in the original firm problem in (9).[30] We find the level of the tax which generates the same level of aggregate output we obtained in the baseline economy with border delays. The results show that the 13.8 days of average border delays we observe in the Sub-Saharan African data are equivalent to a 1.9% tax on investments in foreign materials.

This comparatively low tax rate suggest a very high tax elasticity of output. This is due to the endogenous upgrading choices of firms. The tax on foreign investment only affects type 3 firms, lowering their value. This reduces the fraction of type 3 firms in equilibrium and affects output through the selection channel. Holding the selection margin constant, a tax rate of 3.6% on foreign investments would generate the equivalent amount of average type 3 output.

In Section 3 we derived the user cost of foreign inputs as $r_x = \frac{1-\beta\kappa\psi}{\beta(1-\kappa)}$, i.e., as a decreasing function in the probability of successful delivery, ψ. Without delays but in the presence of a tax on investments in foreign materials, the user cost can be written as $\tilde{r}_x = \frac{1+\tau-\beta\kappa}{\beta(1-\kappa)}$. Given the calibrated value of $\psi = 0.4$, the tax that would equalize the user cost of foreign materials in the two cases would be 0.12%. The fact that the tax rate of 1.9% reported above is an order of magnitude higher speaks to the fact that removing delays has a much broader impact than just lowering the user cost of foreign materials. It also removes the uncertainty surrounding the choices of capital, labor, and local materials and would increase output, even if the choice of foreign materials were to stay constant.

5. Conclusion

Our analysis suggests that simplifying and streamlining the border procedures for imported inputs in Sub-Saharan Africa should be a high-priority item on the region's agenda.

Using the WBES data, we documented that not only are border delays of inputs pervasive across Sub-Saharan Africa, they are also several orders of magnitudes larger than in richer countries. To assess their effects on firms and on the macroeconomy, we developed a dynamic general equilibrium model of heterogeneous firms. In the model, firms differ in productivity, their technology choice, and their levels of local and foreign materials used in production. Importantly, not all firms are subject to delays because smaller, less productive firms opt for a production technology that only uses capital, labor, as well as local materials. Firms that choose to import foreign materials face a probability that those imports are delayed, leading to a suboptimal allocation of factors. Firms can mitigate this risk by holding inventories of foreign materials.

We find that when border delays are eliminated, aggregate output increases by 9.4%. These effects are mainly explained through selection and reallocation. The delays represent a distortion that is positively correlated with firms' productivity, affecting mainly the largest, most productive firms in the economy. Once the delays are removed, more firms choose to import foreign materials and operate a more advanced technology. In this way, factors are reallocated towards more productive firms.

We see two potential avenues for future work. First, one could correlate measures of distortions with measures of firm productivity and use the model to assess the effect of removing import delays on this correlation. The lack of better firm-level data, especially on capital, has prevented us from pursuing this path. Second, in our analysis we assumed a common production function for each firm type. Arguably, the production functions used in our economic model are a better representation of manufacturing firms than service firms. Our choice was partly motivated by the better data availability for the manufacturing sector. While our robustness analysis concluded that import delays are also prevalent in non-manufacturing sector, studying the heterogeneous effects of input delays on sectors of production was beyond the scope of this paper and remains a potential avenue for future research.

Data availability

Replication package already submitted

Appendix A. Data

This appendix includes descriptive statistics and explains key variables in more detail.

WBES questions used To measure the use of foreign inputs by firms, to classify firms into types, and to measure border delays, firm employment and inventories, we rely on the following questions from the World Bank Enterprise Surveys.

[30] The derivations of key equations are shown in Appendix B.5.

Table A.1
Descriptive statistics 1/2.

Country	Uses foreign inputs			Share of foreign inputs			Border delay			Full time employees			Log real GDP
	mean	sd	N	mean	sd	N	mean	sd	N	mean	sd	N	(2010)
Angola	0.63	0.48	139	43.40	25.93	88	11.43	10.66	42	34.27	49.17	349	25.26
Benin	0.69	0.47	148	76.78	30.37	106	14.06	15.08	77	45.44	107.51	150	22.93
Botswana	0.87	0.34	87	71.08	30.90	77	4.76	6.23	61	66.78	231.55	267	23.16
Burkina Faso	0.76	0.43	92	68.05	29.52	70	16.38	26.68	29	28.69	121.11	391	22.93
Burundi	0.79	0.41	60	63.11	33.04	45	32.39	30.51	29	38.47	78.67	157	21.72
Cameroon	0.63	0.48	350	52.03	33.45	209	23.39	25.77	70	23.17	58.87	360	23.91
Cabo Verde	0.58	0.50	74	81.37	28.25	48	20.51	15.35	30	20.37	46.72	152	21.12
Central Afr. Rep.	0.72	0.46	29	74.53	26.20	21	11.86	11.88	9	27.13	47.88	149	21.57
Chad	0.58	0.50	145	71.46	31.38	89	30.76	31.76	50	13.51	27.89	153	22.88
Côte d'ivoire	0.53	0.50	341	64.40	34.66	170	27.06	21.77	83	32.60	91.62	349	24.24
Djibouti	0.69	0.47	52	89.90	20.16	36	5.09	2.66	26	36.25	86.66	261	21.47
Eritrea	0.43	0.50	90	53.44	34.82	38	20.12	22.42	11	17.49	20.17	177	21.45
Ethiopia	0.38	0.49	363	46.68	30.69	166	19.49	16.09	126	37.88	132.80	832	24.41
Gabon	0.85	0.36	168	74.23	29.59	145	12.59	23.38	52	43.29	135.84	170	23.13
Gambia	0.63	0.48	150	75.21	30.22	87	21.39	21.55	45	17.29	34.51	151	21.02
Ghana	0.69	0.46	372	69.65	31.95	251	14.79	13.04	103	34.71	78.94	718	24.24
Guinea	0.74	0.44	137	84.35	23.66	105	18.09	17.20	92	20.39	51.29	149	22.67
Kenya	0.48	0.50	989	58.28	30.99	451	22.52	20.59	250	33.53	154.86	1000	24.61
Lesotho	0.60	0.49	125	69.71	33.74	82	4.42	6.87	72	60.15	1437.07	147	21.41
Liberia	0.46	0.50	150	75.34	28.60	73	8.08	6.07	31	27.05	41.76	151	21.63
Madagascar	0.32	0.47	394	68.29	37.41	139	15.50	19.25	79	67.70	192.70	515	23.02
Malawi	0.81	0.39	149	60.71	30.83	119	20.01	22.83	67	68.97	260.31	498	22.38
Mali	0.76	0.43	185	62.42	32.90	143	31.52	44.85	58	46.42	134.75	183	23.12
Mauritania	0.73	0.45	47	66.81	33.61	35	26.72	19.06	27	40.60	64.64	146	22.32
Mauritius	0.57	0.50	142	76.95	29.99	90	11.70	14.72	62	42.96	152.06	398	23.00
Mozambique	0.46	0.50	597	61.14	34.31	259	21.58	25.85	177	40.45	98.61	601	23.15

Notes.– See notes under Table A.2.

D.12: "*In the last fiscal year, what percentage of this establishment's purchases of material inputs or supplies were: D12.a.) of domestic origin? D12.b.) of foreign origin?*"

D.13: *"Were any of the material inputs or supplies purchased in fiscal year imported directly?"*

D.14: *"In the last complete fiscal year, when this establishment imported material inputs or supplies, how many days did it take on average from the time these goods arrived to their point of entry (e.g. port, airport) until the time these goods could be claimed from customs?"*

D.16: *"At the present time, when this establishment receives delivery of its most important input, on average, how many days of inventory, measured in days of production, does this establishment keep?"*

L.1: *"At the end of the last fiscal year, how many permanent, full-time individuals worked in this establishment?"*

We use the answer to question D.13 to identify type 3 firms, i.e., those firms that import foreign inputs directly. From the answers to D.12 we are able to infer the foreign input share. The answers to question D.14 constitute our measure of border delays. Firm-level employment is taken from question L.1. To measure inventories we rely on earlier WBES surveys that included question D.16. The question does not specifically ask about inventories of foreign inputs. For firms that were classified as type 3 firms and whose "most important input" is sourced locally, the actual amount of inventory of foreign materials may be lower. We therefore interpret our calibration target of matching the average days of inventories of foreign input held as an upper bound of the importance of inventories and interpret our quantitative results as conservative.

Descriptive statistics In Tables A.1 and A.2 we report summary statistics of our main variables. For each country in our sample, the tables include the fraction of firms that uses foreign inputs, the share of foreign inputs (in %), the average border delays, the number of employees, and the country's 2010-level of real GDP in logs. While reported in the tables, we exclude South African data from our empirical analysis because the last survey was conducted in 2020 and 2021 and is thus potentially heavily affected by the global Covid-19 pandemic. Statistics descriptive related to inventory keeping are provided in Table A.3

Alternative firm populations In our analysis and calibration, we used data moments for all firms in the WBES data, which includes all (formal) business in manufacturing and most non-manufacturing sectors. Additionally, the baseline results use information on firms of all sizes. In Table A.4 we show robustness with respect to those two dimensions. The first column shows data moments from our baseline sample used in the main text (the same moments reported in Table 3).

First, we split the sample into manufacturing and non-manufacturing firms. We find that while manufacturing firms are larger on average, the distribution across firm types as well as employment concentration moments are very similar across the two groups. Most importantly, we find that average import delays are not larger for manufacturing firms. On the contrary, our data shows a higher average delay for the service sector, which also has a slightly larger share of foreign materials. A caveat is the better data availability for manufacturing firms. Of the 42 countries included in our sample, only 20 of them report useable data for non-manufacturing firms.

Table A.2
Descriptive statistics 2/2.

Country	Uses foreign inputs			Share of foreign inputs			Border delay			Full time employees			Log real GDP
	mean	sd	N	mean	sd	N	mean	sd	N	mean	sd	N	(2010)
Namibia	0.45	0.50	153	51.25	38.23	89	5.44	9.27	66	22.83	50.82	568	22.90
Niger	0.57	0.50	132	74.80	26.04	78	28.25	31.66	44	35.98	82.35	150	22.71
Nigeria	0.34	0.47	972	48.19	26.95	398	9.00	14.27	142	20.78	107.67	2564	26.67
Rwanda	0.62	0.49	74	58.39	36.88	52	14.27	25.69	38	36.20	100.09	238	22.52
Senegal	0.41	0.49	234	44.73	31.81	79	17.93	17.84	31	58.08	214.85	596	23.40
Sierra Leone	0.48	0.50	152	62.31	33.69	90	12.48	8.60	29	14.48	21.37	152	21.96
Somalia	0.86	0.34	247	58.62	31.51	197	6.85	12.05	74	20.08	32.84	250	22.06
South Africa	0.08	0.27	1071	44.66	21.56	114	10.67	13.36	44	51.50	277.71	1093	26.38
South Sudan	0.87	0.33	89	65.51	27.25	77	15.10	19.99	40	13.09	22.84	733	23.83
Sudan	0.66	0.48	101	53.77	26.95	67	6.40	2.80	28	26.68	39.01	660	24.71
Eswatini	0.46	0.50	102	64.79	30.76	53	4.04	4.55	66	35.72	85.45	149	21.98
Tanzania	0.63	0.48	355	52.11	30.33	244	29.13	18.70	71	24.23	104.76	718	24.27
Togo	0.60	0.49	149	71.92	31.46	91	30.56	34.59	75	48.83	104.43	150	21.86
Uganda	0.40	0.49	368	33.86	28.84	145	19.44	16.53	56	19.53	116.60	740	23.93
Zambia	0.49	0.50	354	54.10	27.29	157	14.47	12.79	106	24.95	54.50	714	23.53
Zimbabwe	0.66	0.47	592	58.14	32.93	374	8.99	11.05	247	31.88	434.95	600	23.37

Notes.– The columns labeled "Uses foreign inputs" report answers to *D12.b*, assigning a value of 1 to responses larger than zero and 0 otherwise. The columns "Share of foreign inputs" report answers to *D12.b* for firms with $D12.b > 0$. The shares are expressed in %. The columns "Border delay" and "Full time employees" respectively report answers to *D.14* and *L.1*. Source: World Bank Enterprise Survey, World Bank World Development Indicators, and authors' calculations.

Table A.3
Inventories held by type 3 firms (in number of days of production).

Country	Year	N	Med	Mean	SD
Angola	2006	425	15.00	22.19	20.31
Botswana	2006	114	21.00	29.93	40.44
Burkina Faso	2006	51	1.00	24.90	53.26
Burundi	2006	102	45.00	96.42	139.82
Cameroon	2006	119	45.00	61.06	76.43
Cameroon	2009	363	30.00	45.65	67.69
Cabo Verde	2006	47	30.00	35.68	55.50
Cote d'Ivoire	2009	526	60.00	45.92	38.09
Ethiopia	2011	644	60.00	75.47	72.74
Gambia	2006	33	42.00	50.30	41.63
Ghana	2007	616	7.00	17.36	18.13
Guinea	2006	135	15.00	25.01	31.18
Guinea Bissau	2006	50	13.00	14.12	9.26
Kenya	2007	657	30.00	63.11	70.22
Kenya	2013	781	30.00	38.83	42.18
Madagascar	2005	293	60.00	84.35	107.77
Madagascar	2009	445	30.00	50.84	85.70
Malawi	2005	160	60.00	68.64	76.85
Mali	2007	490	10.00	17.53	21.61
Mali	2010	360	3.00	21.29	53.70
Mauritania	2006	80	30.00	42.74	36.26
Mauritius	2005	212	30.00	52.06	54.80
Mozambique	2007	599	15.00	23.08	27.89
Namibia	2006	106	28.00	32.97	38.35
Niger	2005	125	30.00	55.15	83.06
Nigeria	2007	2387	14.00	28.39	37.58
Rwanda	2006	59	4.00	18.49	40.18
Senegal	2007	506	15.00	23.74	26.60
South Africa	2007	1057	15.00	25.30	24.74
Swaziland	2006	70	7.00	14.86	41.42
Tanzania	2006	273	30.00	45.57	38.86
Uganda	2006	307	28.00	36.67	32.06
Zambia	2007	603	14.00	27.05	50.09
Zimbabwe	2011	600	30.00	53.09	46.02

As a second robustness check, we consider only those firms that have at least five employees. The rationale for this is that the WBES data is only designed to be representative of firms with at least five employees, although it includes a sizeable number of firms with fewer than five employees. In the baseline sample, firms with less than five employees make up about 7% of all firms, and 4% of firms that directly import foreign inputs (type 3 firms). In the last column of Table A.4, we report our targeted data moments for

Table A.4
Data moments for alternative firm populations.

Data moment	All firms	Manufacturing	Non-manu	$n \geq 5$
Average employment of type 1 firms	23.27	27.04	17.56	24.11
Average employment of type 2 firms	27.69	34.84	22.90	28.52
Average employment of type 3 firms	89.29	98.99	45.92	90.33
Share of large firms (100+)	0.07	0.08	0.05	0.09
Employment share of large firms (100+)	0.47	0.48	0.42	0.55
Fraction of type 1 firms	0.49	0.48	0.42	0.48
Fraction of type 2 firms	0.27	0.27	0.26	0.27
Employment share of type 1 firms	0.31	0.30	0.30	0.30
Average import delay of (days)	13.76	13.34	21.39	13.83
Foreign materials share	0.58	0.57	0.75	0.58

Notes: Column "Manufacturing" reports moments for manufacturing firms and column "Non-manu" reports for non-manufacturing firms. The last column reports moments for firms with at least five employees.

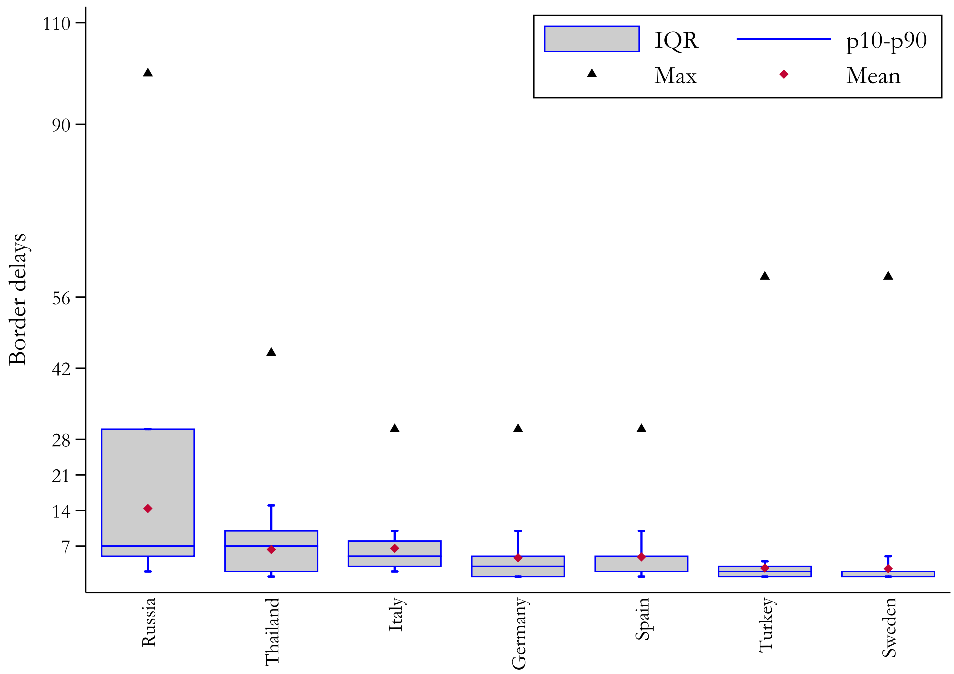

Fig. A.1. Border delays in other countries. Notes.– The figure shows country-specific distributions of border delays in days. The box plots cover the 25th to 75th percentiles of delays. Medians are indicated by the horizontal bar. The lines extending from the boxes show the 10th and 90th percentiles. Averages are shown as red diamonds. The black triangles show the maximum delays. Minimum delays were zero in all cases. Source: World Bank Enterprise Survey and authors' calculations.

the sample of firms with at least five employees. The results are very similar to our baseline numbers used in the main text, reported in the column "All firms". In particular, the average import delay is virtually unchanged.

Border delays in non-SSA countries In Fig. A.1 we replicate Fig. 2 for Russia, Thailand, Italy, Germany, Spain, Turkey and Sweden. The differences are stark. Although the maximum delay in Russia is comparable to that observed in some Sub-saharan African countries, the overall distribution is more concentrated towards low delays in all these countries. For comparison, average delays are 2.6 days in Sweden and Turkey, under five days in Spain and Germany, and around 6 to 7 days in Thailand, Italy and Belgium. These are 4 to 15 times lower than the average delay in Burundi, Mali or Togo.

Appendix B. Model

B.1. Type 1 firm: optimal choices

A type 1 firm produces using only capital and labor, *cf.* (18). The first-order condition of (20) with respect to labor is given by

$$\varepsilon k^{\alpha_1} n^{\alpha_2 - 1} \alpha_2 - w = 0. \tag{B.1}$$

The optimality condition for capital is given by

$$\varepsilon k^{\alpha_1-1}n^{\alpha_2}\alpha_1 - r = 0, \tag{B.2}$$

with the user cost of capital given by

$$r = \frac{1-\beta\kappa}{\beta(1-\kappa)} - (1-\delta). \tag{B.3}$$

Combining the two first-order conditions gives

$$\frac{w}{r} = \frac{\varepsilon k^{\alpha_1}n^{\alpha_2-1}\alpha_2}{\varepsilon k^{\alpha_1-1}n^{\alpha_2}\alpha_1} = \frac{k\alpha_2}{n\alpha_1} \quad \Leftrightarrow \quad n = k\cdot\left(\frac{\alpha_2}{w}\right)\left(\frac{r}{\alpha_1}\right). \tag{B.4}$$

Total production $\varepsilon F(k,n)$ can then be written as

$$\varepsilon k^{\alpha_1}k^{\alpha_2}\cdot\left(\frac{\alpha_2}{w}\right)^{\alpha_2}\left(\frac{r}{\alpha_1}\right)^{\alpha_2} = A_1 k^{\gamma}, \tag{B.5}$$

where we have defined the following terms

$$\gamma = \alpha_1 + \alpha_2 \tag{B.6}$$

$$A_1 = \varepsilon\cdot\left(\frac{\alpha_2}{w}\right)^{\alpha_2}\left(\frac{r}{\alpha_1}\right)^{\alpha_2}. \tag{B.7}$$

Output plus the value of non-depreciated capital minus the wage costs becomes

$$A_1k^{\gamma} + k\left((1-\delta) - \alpha_2\left(\frac{r}{\alpha_1}\right)\right). \tag{B.8}$$

The firm value (20) can be reformulated with only one choice variable, capital k:

$$V_1(\varepsilon) = \max_k -k + \beta(1-\kappa)\left[A_1k^{\gamma} + k\left((1-\delta)-\alpha_2\left(\frac{r}{\alpha_1}\right)\right) + \mathbb{E}_{\varepsilon'|\varepsilon}V_1(\varepsilon')\right] + \beta\kappa k, \tag{B.9}$$

with associated first-order condition for capital

$$-1 + \beta(1-\kappa)\left[A_1\gamma k^{\gamma-1} + (1-\delta) - \alpha_2\left(\frac{r}{\alpha_1}\right)\right] + \beta\kappa = 0, \tag{B.10}$$

implying the optimality condition

$$r = \alpha_1 A_1 k^{\gamma-1} \quad \Leftrightarrow \quad k_1^* = A_1^{\frac{1}{1-\gamma}}\left(\frac{\alpha_1}{r}\right)^{\frac{1}{1-\gamma}}. \tag{B.11}$$

Production in (B.5) becomes

$$A_1k^{\gamma} = A_1^{\frac{1}{1-\gamma}}\left(\frac{\alpha_1}{r}\right)^{\frac{\gamma}{1-\gamma}}. \tag{B.12}$$

The firm value (B.9) can be written as

$$V_1(\varepsilon) = \beta(1-\kappa)\left(A_1^{\frac{1}{1-\gamma}}\left(\frac{\alpha_1}{r}\right)^{\frac{\gamma}{1-\gamma}}(1-\gamma) + \mathbb{E}_{\varepsilon'|\varepsilon}W_1(\varepsilon')\right). \tag{B.13}$$

From (B.4) and (B.11) optimal labor demand is given by

$$n_1^* = A_1^{\frac{1}{1-\gamma}}\left(\frac{\alpha_1}{r}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{\alpha_2}{w}\right). \tag{B.14}$$

B.2. Type 2 firm: optimal choices

The production function is given by (13). The first-order condition of the value of a not-upgraded type 2 firm, (15), with respect to labor is given by

$$\varepsilon\left(\phi(k^{\alpha_1}n^{\alpha_2})^{\rho_f} + (1-\phi)m^{\rho_f}\right)^{1/\rho_f-1}\phi(k^{\alpha_1}n^{\alpha_2})^{\rho_f-1}k^{\alpha_1}n^{\alpha_2-1}\alpha_2 - w = 0 \tag{B.15}$$

The optimality condition for capital is given by

$$\varepsilon\left(\phi(k^{\alpha_1}n^{\alpha_2})^{\rho_f} + (1-\phi)m^{\rho_f}\right)^{1/\rho_f-1}\phi(k^{\alpha_1}n^{\alpha_2})^{\rho_f-1}k^{\alpha_1-1}n^{\alpha_2}\alpha_1 - r = 0, \tag{B.16}$$

with the user cost of capital defined in (B.3). Using the two first-order conditions together implies that labor can again be written as a linear function of capital:

$$n = k \cdot \left(\frac{\alpha_2}{w}\right)\left(\frac{r}{\alpha_1}\right) \tag{B.17}$$

The capital-labor bundle then becomes $k^{\alpha_1} n^{\alpha_2} = k^{\gamma} \cdot \left(\frac{\alpha_2}{w}\right)^{\alpha_2}\left(\frac{r}{\alpha_1}\right)^{\alpha_2}$, with γ defined in (B.6). The production function (13) can then be written as a function of k and m only:

$$\tilde{F}(k,m) = \left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)m^{\rho_f}\right)^{1/\rho_f}, \tag{B.18}$$

with

$$A_2 = \phi \cdot \left(\frac{\alpha_2}{w}\right)^{\alpha_2\cdot\rho_f}\left(\frac{r}{\alpha_1}\right)^{\alpha_2\cdot\rho_f} \tag{B.19}$$

Output plus the value of non-depreciated capital minus the wage bill becomes

$$\varepsilon \tilde{F}(k,m) - wn + (1-\delta)k = \varepsilon \tilde{F}(k,m) + k\left((1-\delta) - \alpha_2\left(\frac{r}{\alpha_1}\right)\right), \tag{B.20}$$

and the firm value (15) can be written as

$$V_2(\varepsilon) = \max_{k,m} \beta(1-\kappa)\left[\varepsilon\tilde{F}(k,m) - k\cdot r\frac{\gamma}{\alpha_1} + \mathbb{E}_{\varepsilon'|\varepsilon} W_2(\varepsilon')\right] - m(1-\beta\kappa). \tag{B.21}$$

The associated first-order conditions for capital and local materials are

$$r\cdot\frac{\gamma}{\alpha_1} = \varepsilon\frac{1}{\rho_f}\left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)m^{\rho_f}\right)^{1/\rho_f - 1}\cdot\gamma\rho_f A_2 k^{\gamma\cdot\rho_f-1} \tag{B.22}$$

and

$$r_m = \varepsilon\frac{1}{\rho_f}\left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)m^{\rho_f}\right)^{1/\rho_f - 1}\cdot\rho_f(1-\phi)m^{\rho_f-1}, \tag{B.23}$$

where the user cost of local materials is

$$r_m = \frac{1-\beta\kappa}{\beta(1-\kappa)}. \tag{B.24}$$

Using the two first-order conditions together implies

$$\frac{r\cdot\frac{\gamma}{\alpha_1}}{r_m} = \frac{\gamma\rho_f A_2 k^{\gamma\cdot\rho_f-1}}{\rho_f(1-\phi)m^{\rho_f-1}} \quad\Leftrightarrow\quad m = \left(\frac{(1-\phi)r}{A_2\alpha_1 r_m}\right)^{\frac{1}{1-\rho_f}}\cdot k^{\frac{1-\gamma\cdot\rho_f}{1-\rho_f}}. \tag{B.25}$$

Then production in (B.18) can be written as a function of capital only

$$\hat{F}(k) = \left(A_2 k^{\gamma\cdot\rho_f} + B_2 k^{\eta\cdot\rho_f}\right)^{1/\rho_f}, \tag{B.26}$$

where

$$B_2 = (1-\phi)^{\frac{1}{1-\rho_f}}\cdot\left(\frac{r}{A_2\alpha_1 r_m}\right)^{\frac{\rho_f}{1-\rho_f}}, \tag{B.27}$$

and

$$\eta = \frac{1-\gamma\cdot\rho_f}{1-\rho_f}. \tag{B.28}$$

The value function (B.21) can then be formulated with only one choice variable

$$V_2(\varepsilon) = \max_{k} \beta(1-\kappa)\left(\varepsilon\hat{F}(k) - k\cdot r\frac{\gamma}{\alpha_1} - k^{\eta}\cdot\frac{B_2}{A_2}\frac{r}{\alpha_1} + \mathbb{E}_{\varepsilon'|\varepsilon}W_2(\varepsilon')\right), \tag{B.29}$$

with associated first-order condition for capital

$$0 = \varepsilon\left(A_2 k^{\gamma\cdot\rho_f} + B_2 k^{\eta\cdot\rho_f}\right)^{1/\rho_f-1}\cdot\left(A_2\gamma k^{\gamma\cdot\rho_f-1} + B_2\eta k^{\eta\cdot\rho_f-1}\right) - \frac{r}{\alpha_1}\left(\gamma + \eta\frac{B_2}{A_2}k^{\eta-1}\right). \tag{B.30}$$

The first two terms represent the change in the marginal product, while the last term represents the total marginal cost of investing an additional unit of capital. It is composed of the user-cost of capital, as well as the resulting marginal changes in the costs of labor and local materials.

The solution to (B.29) is k_2^*, which is found numerically. Then we use (B.17) and (B.25) to back out the optimal policy functions for labor, n_2^*, and local materials, $m_2{}^*$.

The optimal cutoff value $\bar{\xi}_2$ in (17) follows from taking the first order condition of (15) with respect to $\bar{\xi}_2$ and applying the Leibniz formula for the derivative of an integral with functional bounds,

$$\frac{\partial}{\partial \alpha}\int_{a(\alpha)}^{b(\alpha)} h(y,\alpha)dy = h\left(b(\alpha),\alpha\right)\cdot\frac{\partial b(\alpha)}{\partial \alpha} - h\left(a(\alpha),\alpha\right)\cdot\frac{\partial a(\alpha)}{\partial \alpha} + \int_{a(\alpha)}^{b(\alpha)}\frac{\partial h(y,\alpha)}{\partial \alpha}dy.$$

B.3. Type 3 firm: derivations

Laws of motion in recursive form The laws of motion for the endogenous state variables o_x and i_x in recursive form are given by

$$o'_x = \begin{cases} 0 & \text{with probability } \psi \\ o_x + \chi & \text{with probability } 1-\psi \end{cases} \tag{B.31}$$

and

$$i'_x = \begin{cases} \left(i_x + o_x + \chi\right)\cdot(1-\omega) & \text{with probability } \psi \\ i_x\cdot(1-\omega) & \text{with probability } 1-\psi \end{cases}, \tag{B.32}$$

whereas available foreign materials for production in the following production phase are given by

$$\imath'_x = \begin{cases} i_x + o_x + \chi & \text{with probability } \psi \\ i_x & \text{with probability } 1-\psi. \end{cases} \tag{B.33}$$

First-order conditions for type 3 firm The first-order conditions of (9) with respect to n, k, and m are given by (10). Using the partial derivatives of the production function with respect to n and k, one can write labor as a linear function of capital, $n = k\cdot\left(\frac{\alpha_2}{w}\right)\left(\frac{r}{\alpha_1}\right)$, as in (B.17). This can be used to rewrite the capital-labor bundle as $A_2 k^{\gamma\cdot\rho_f}$, with A_2 defined in (B.19). The production function (1) can then be written as

$$\tilde{F}(k,m,x) = \left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)M(m,x)^{\rho_f}\right)^{1/\rho_f}, \tag{B.34}$$

and the firm value in (9) can be written as

$$\begin{aligned} W_3(\varepsilon, i_x, o_x) = \max_{k,m,\chi}\Bigg\{ & -\chi - \theta i_x + \\ & \beta(1-\kappa)\left[\max_{\omega}\mathbb{E}_{\psi}\left\{\varepsilon\tilde{F}(k,m,x) - k\cdot r\frac{\gamma}{\alpha_1} - m\cdot r_m + \mathbb{E}_{\varepsilon'|\varepsilon}W_3(\varepsilon', i'_x, o'_x)\right\}\right] + \\ & \beta\kappa\mathbb{E}_{\psi}\left(\imath'_x\right)\Bigg\}. \end{aligned} \tag{B.35}$$

The user-costs of capital and local materials were defined in (B.3) and (B.24). The decision ω, i.e., what fraction of available foreign materials to use in production, is made *after* the realization of the delivery risk. We denote as ω_1 and ω_0 the choices in case of delivery or no delivery. The first-order condition of (B.35) with respect to capital is given by

$$r\frac{\gamma}{\alpha_1} = \psi\varepsilon\frac{\partial\tilde{F}(k,m,\omega_1\left(i_x+o_x+\chi\right))}{\partial k} + (1-\psi)\varepsilon\frac{\partial\tilde{F}(k,m,\omega_0\cdot i_x)}{\partial k}, \tag{B.36}$$

where we made the input x into the production function (B.34) explicit. In case of delivery, which occurs with probability ψ, available foreign materials are $i_x + o_x + \chi$ and the firm optimally chooses to use a fraction ω_1 of them. In case of no delivery, available foreign materials are i_x, of which a fraction ω_0 are used. The partial derivative of production with respect to capital is

$$\frac{\partial\tilde{F}(k,m,x)}{\partial k} = \left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)M(m,x)^{\rho_f}\right)^{1/\rho_f - 1}\cdot A_2\gamma k^{\gamma\cdot\rho_f - 1} \tag{B.37}$$

The first-order condition of (B.35) with respect to local materials is analogously given by

$$r_m = \psi\varepsilon\frac{\partial\tilde{F}(k,m,\omega_1\left(i_x+o_x+\chi\right))}{\partial m} + (1-\psi)\varepsilon\frac{\partial\tilde{F}(k,m,\omega_0\cdot i_x)}{\partial m}, \tag{B.38}$$

with

$$\frac{\partial\tilde{F}(k,m,x)}{\partial m} = \left(A_2 k^{\gamma\cdot\rho_f} + (1-\phi)M(m,x)^{\rho_f}\right)^{1/\rho_f - 1} \tag{B.39}$$

$$\times(1-\phi)M(m,x)^{\rho_f-1}\cdot\left(\phi_m m_t^{\rho_m} + (1-\phi_m)x_t^{\rho_m}\right)^{1/\rho_m - 1}$$

$$\times\,\phi_m m_t^{\rho_m - 1}. \tag{B.40}$$

B.4. Frictionless model

The model without input delays can be written as follows. For the type 1 and type 2 firms, the setup is identical to that derived in Appendices B.1 and B.2. For type 3 firms, because orders are never outstanding and there is no need for inventories, the only state variable is now future productivity ε. The problem is as follows.

$$W_3(\varepsilon) = \max_{k,n,m,x} (k+m+x)(-1+\beta\kappa) + \beta(1-\kappa)\left[\varepsilon F(k,n,m,x) - wn + (1-\delta)k + \mathbb{E}_{\varepsilon'|\varepsilon} W_3(\varepsilon')\right], \tag{B.41}$$

with the production function given by (1). Note that ω is no longer a choice variable. In addition, the firm directly chooses the amount of foreign material to use in production, x. We first determine the optimal mix between local and foreign materials. The first-order condition of (B.41) with respect to m and x are given by

$$r_m = \varepsilon \frac{\partial F}{\partial m} \qquad r_m = \varepsilon \frac{\partial F}{\partial x}, \tag{B.42}$$

where the user cost of local materials is given by (B.24). The partial derivatives are

$$\begin{aligned}\frac{\partial F}{\partial m} &= \left(\phi (k^{\alpha_1} n^{\alpha_2})^{\rho_f} + (1-\phi) M(m,x)^{\rho_f}\right)^{1/\rho_f - 1} \times \\ &\quad (1-\phi) M(m,x)^{\rho_f - 1} \left(\phi_m m^{\rho_m} + (1-\phi_m) x^{\rho_m}\right)^{1/\rho_m - 1} \phi_m m^{\rho_m - 1}\end{aligned} \tag{B.43}$$

and

$$\begin{aligned}\frac{\partial F}{\partial x} &= \left(\phi (k^{\alpha_1} n^{\alpha_2})^{\rho_f} + (1-\phi) M(m,x)^{\rho_f}\right)^{1/\rho_f - 1} \times \\ &\quad (1-\phi) M(m,x)^{\rho_f - 1} \left(\phi_m m^{\rho_m} + (1-\phi_m) x^{\rho_m}\right)^{1/\rho_m - 1} (1-\phi_m) x^{\rho_m - 1}.\end{aligned} \tag{B.44}$$

Using the two first-order conditions for m and x yields:

$$m = \left(\frac{\phi_m}{1-\phi_m}\right)^{\frac{1}{1-\rho_m}} \cdot x \tag{B.45}$$

We can then write the material bundle $M(m,x)$ in (2) as

$$x \cdot \left(\phi_m \left(\frac{\phi_m}{1-\phi_m}\right)^{\frac{\rho_m}{1-\rho_m}} + (1-\phi_m)\right)^{1/\rho_m}. \tag{B.46}$$

In a second step, we determine optimal labor as a function of capital. This follows the same steps as those in Appendix B.1 and results in $n = k \cdot \left(\frac{\alpha_2}{w}\right)\left(\frac{r}{\alpha_1}\right)$. This can be used to rewrite the production function (1) as a function of capital and foreign materials only:

$$F(k, n(k), m(x), x) = \left(A_2 k^{\gamma \cdot \rho_f} + A_3 x^{\rho_f}\right)^{1/\rho_f}, \tag{B.47}$$

with γ defined in (B.6), A_2 defined in (B.19), and

$$A_3 = (1-\phi)\left(\phi_m \left(\frac{\phi_m}{1-\phi_m}\right)^{\frac{\rho_m}{1-\rho_m}} + (1-\phi_m)\right)^{\rho_f/\rho_m}. \tag{B.48}$$

The firm value (B.41) can then be written as

$$\begin{aligned}W_3(\varepsilon) = \max_{k,x}\ & x\left(1 + \left(\frac{\phi_m}{1-\phi_m}\right)^{\frac{1}{1-\rho_m}}\right)(-1+\beta\kappa) + \\ & \beta(1-\kappa)\left[\varepsilon \left(A_2 k^{\gamma\cdot\rho_f} + A_3 x^{\rho_f}\right)^{1/\rho_f} - k\gamma \frac{r}{\alpha_1} + \mathbb{E}_{\varepsilon'|\varepsilon} W_3(\varepsilon')\right],\end{aligned} \tag{B.49}$$

with associated first-order condition for capital

$$r \cdot \frac{\gamma}{\alpha_1} = \varepsilon \frac{1}{\rho_f}\left(A_2 k^{\gamma\cdot\rho_f} + A_3 x^{\rho_f}\right)^{1/\rho_f - 1} \cdot \gamma \rho_f A_2 k^{\gamma\cdot\rho_f - 1} \tag{B.50}$$

and for foreign materials

$$r_M = \varepsilon \frac{1}{\rho_f}\left(A_2 k^{\gamma\cdot\rho_f} + A_3 x^{\rho_f}\right)^{1/\rho_f - 1} \cdot \rho_f A_3 x^{\rho_f - 1}, \tag{B.51}$$

where the effective user cost of foreign materials, including the cost of local materials, is

$$r_M = \frac{(1-\beta\kappa)}{\beta(1-\kappa)} \cdot \left(1 + \left(\frac{\phi_m}{1-\phi_m} \right)^{\frac{1}{1-\rho_m}} \right). \tag{B.52}$$

Using the two first-order conditions together implies

$$\frac{r \cdot \frac{\gamma}{\alpha_1}}{r_M} = \frac{\gamma A_2 k^{\gamma\cdot\rho_f - 1}}{A_3 x^{\rho_f - 1}} \quad \Leftrightarrow \quad k = \left(\frac{A_2 \alpha_1 r_M}{A_3 r} \right)^{\frac{1}{1-\gamma\cdot\rho_f}} \cdot x^{\frac{1-\rho_f}{1-\gamma\cdot\rho_f}}. \tag{B.53}$$

The production function (B.47) becomes

$$F(k(x), n(k(x)), m(x), x) = \left(A_4 x^{\nu\cdot\rho_f} + A_3 x^{\rho_f} \right)^{1/\rho_f}, \tag{B.54}$$

where

$$A_4 = A_2^{\frac{1}{1-\gamma\cdot\rho_f}} \cdot \left(\frac{r_M \alpha_1}{A_3 r} \right)^{\frac{\gamma\cdot\rho_f}{1-\gamma\cdot\rho_f}} \tag{B.55}$$

and

$$\nu = \frac{\gamma(1-\rho_f)}{1-\gamma\cdot\rho_f}. \tag{B.56}$$

The value function (B.49) can then be written as

$$\begin{aligned} W_3(\varepsilon) = \max_x \beta(1-\kappa) \Bigg[& \varepsilon \left(A_4 x^{\nu\cdot\rho_f} + A_3 x^{\rho_f} \right)^{1/\rho_f} - \\ & \left(\frac{A_2 r_M}{A_3} \right)^{\frac{1}{1-\gamma\cdot\rho_f}} \gamma \left(\frac{r}{\alpha_1} \right)^{\frac{-\gamma\rho_f}{1-\gamma\rho_f}} \cdot x^{\frac{1-\rho_f}{1-\gamma\cdot\rho_f}} - x \cdot r_M + \mathbb{E}_{\varepsilon'|\varepsilon} W_3(\varepsilon') \Bigg] \end{aligned} \tag{B.57}$$

with associated first-order condition for foreign materials x

$$\begin{aligned} 0 = & -\left(\frac{A_2 r_M}{A_3} \right)^{\frac{1}{1-\gamma\cdot\rho_f}} \gamma \left(\frac{r}{\alpha_1} \right)^{\frac{-\gamma\rho_f}{1-\gamma\rho_f}} \cdot \frac{1-\rho_f}{1-\gamma\cdot\rho_f} x^{\frac{1-\rho_f}{1-\gamma\cdot\rho_f} - 1} - r_M + \\ & \varepsilon \left(A_4 x^{\nu\cdot\rho_f} + A_3 x^{\rho_f} \right)^{1/\rho_f - 1} \left(\nu A_4 x^{\nu\rho_f - 1} + A_3 x^{\rho_f - 1} \right). \end{aligned} \tag{B.58}$$

The solution implies a value of x_3^*, found numerically. Then we use (B.45), (B.53), and (B.17) to back out the optimal policy functions for local materials, capital, and labor, $m_3{}^*$, k_3^*, and n_3^*.

B.5. Model with tax on investment

The setup is very close to the frictionless setup in Appendix B.4. We only highlight the differences here. For type 3 firms the problem is as follows. The tax rate is given by τ.

$$\begin{aligned} W_3(\varepsilon) = \max_{k,n,m,x} & (k+m)(-1+\beta\kappa) + x(-(1+\tau)+\beta\kappa) + \\ & \beta(1-\kappa) \left[\varepsilon F(k,n,m,x) - wn + (1-\delta)k + \mathbb{E}_{\varepsilon'|\varepsilon} W_3(\varepsilon') \right]. \end{aligned} \tag{B.59}$$

The first-order condition of (B.59) with respect to x is given by

$$\frac{1+\tau-\beta\kappa}{\beta(1-\kappa)} = \varepsilon \frac{\partial F}{\partial x}, \tag{B.60}$$

which can be used together with the first-order condition for m to write

$$m = \left(\tilde{\tau} \cdot \frac{\phi_m}{1-\phi_m} \right)^{\frac{1}{1-\rho_m}} \cdot x, \tag{B.61}$$

where $\tilde{\tau} = \frac{1+\tau-\beta\kappa}{1-\beta\kappa}$. We can then write the material bundle $M(m,x)$ in (2) as

$$x \cdot \left(\phi_m \left(\tilde{\tau} \cdot \frac{\phi_m}{1-\phi_m} \right)^{\frac{\rho_m}{1-\rho_m}} + (1-\phi_m) \right)^{1/\rho_m}. \tag{B.62}$$

This can be used to rewrite the production function (1) as a function of capital and foreign materials only:

$$F(k, n(k), m(x), x) = \left(A_2 k^{\gamma\cdot\rho_f} + A_3 x^{\rho_f} \right)^{1/\rho_f}, \tag{B.63}$$

Table C.5
Robustness.

Average border delays (in days)	Fixed labor		Higher ϕ_m		$n \geq 5$	
	Delays	No delays	Delays	No delays	Delays	No delays
	13.8	0	13.8	0	13.8	0
Aggregate output	1.00	1.11	1.00	1.05	1.00	1.06
Labor supply	1.00	1.00	1.00	1.00	1.00	0.97
Capital	1.00	1.03	1.00	1.06	1.00	0.99
Local materials	1.00	1.05	1.00	0.95	1.00	1.03
Foreign materials	1.00	1.28	1.00	1.41	1.00	1.21
Outstanding orders	1.00	0	1.00	0	1.00	0
Inventories	1.00	0	1.00	0	1.00	0
Wage rate	1.00	1.03	1.00	1.06	1.00	1.03
Mass of firms	1.00	0.91	1.00	0.90	1.00	0.89
Fraction of type 1 firms	0.49	0.40	0.51	0.36	0.46	0.35
Fraction of type 2 firms	0.25	0.28	0.25	0.30	0.25	0.29
Fraction of type 3 firms	0.26	0.32	0.24	0.34	0.30	0.37
Employment share type 1 firms	0.29	0.18	0.28	0.14	0.26	0.16
Employment share type 2 firms	0.18	0.14	0.17	0.12	0.16	0.13
Employment share type 3 firms	0.54	0.68	0.55	0.75	0.58	0.71

Note: The values for the economy without delays have been normalized to one.

with γ defined in (B.6), A_2 defined in (B.19), and

$$A_3 = (1-\phi)\left(\phi_m\left(\tilde{\tau}\cdot\frac{\phi_m}{1-\phi_m}\right)^{\frac{\rho_m}{1-\rho_m}} + (1-\phi_m)\right)^{\rho_f/\rho_m}. \tag{B.64}$$

The firm value (B.59) can then be written as

$$W_3(\varepsilon) = \max_{k,x} x\left(\left(\tilde{\tau}\cdot\frac{\phi_m}{1-\phi_m}\right)^{\frac{1}{1-\rho_m}}(-1+\beta\kappa) - (1+\tau) + \beta\kappa\right) + \beta(1-\kappa)\left[\varepsilon\left(A_2k^{\gamma\cdot\rho_f} + A_3x^{\rho_f}\right)^{1/\rho_f} - k\gamma\frac{r}{\alpha_1} + \mathbb{E}_{\varepsilon'|\varepsilon}W_3(\varepsilon')\right], \tag{B.65}$$

with associated first-order condition for foreign materials

$$r_M = \varepsilon\frac{1}{\rho_f}\left(A_2k^{\gamma\cdot\rho_f} + A_3x^{\rho_f}\right)^{1/\rho_f-1}\cdot\rho_f A_3x^{\rho_f-1}, \tag{B.66}$$

where the effective user cost of foreign materials, including the cost of local materials, is

$$r_M = r_m\cdot\left(\left(\tilde{\tau}\cdot\frac{\phi_m}{1-\phi_m}\right)^{\frac{1}{1-\rho_m}} + \tilde{\tau}\right). \tag{B.67}$$

This lets use derive an equation for k as a function of x, as in the frictionless case. The resulting only remaining first-order condition for x is then unchanged compared to (B.58), the only differences being that the terms that enter include the effect of taxes through $\tilde{\tau}$.

Appendix C. Robustness analysis

We present three robustness analyses of our main findings in Table C.5. The table has the same structure as Table 4 in the main text. The first robustness exercise studies the case where labor is supplied exogenously. The increase in the wage rate following the removal of border delays does not exert an income effect in this model and labor supply remains unchanged. Relative to our result in the main text this implies that the output effects of eliminating border delays are slightly larger.

The second exercise studies the effect of eliminating border delays in an economy with a lower share of foreign inputs. Due to data limitations, we assumed in the benchmark analysis that for type 3 firms, all inputs of foreign origin are directly imported. However, some of those can be materials of foreign origin that are sourced locally. For the robustness exercise, we increase the coefficient ϕ_m in (2) from 0.4 to 0.6. The latter denotes the mid-point between the value we are using in the baseline analysis and the share of foreign, locally sourced inputs reported by type 2 firms in the data. We find that in that case removing all border delays has a positive output effect of around 5%.

Finally, we include a robustness exercise in which we condition on a minimum size of firms. As was mentioned in the discussion surrounding Table A.4, the WBES data is only meant to be representative of firms with at least five employees. The last column of that table shows the calibration targets if we remove firms with fewer than five employees from the data. To make the model consistent

Table C.6
Calibrated parameters for the $n \geq 5$ case.

Parameter	Value	Moment	Data	Model
μ_ε^1	−2.49	Average employment of type 1 firms	24.11	22.66
μ_ε^2	−0.64	Average employment of type 2 firms	28.52	24.83
μ_ε^3	−0.15	Average employment of type 3 firms	90.33	77.34
ρ_ε	0.71	Share of large firms (100+)	8.76	7.57
σ_ε	0.05	Employment share of large firms (100+)	55.03	48.26
Ξ_1	8.76	Fraction of type 1 firms	48.27	45.60
Ξ_2	11.22	Fraction of type 2 firms	27.01	24.81
c_e	148.08	Employment share of type 1 firms	30.45	26.24
θ	0.03	Average days worth of inventory held	31.31	29.79

with this data treatment, we re-calibrate the model by computing model moments only for firms with at least five employees. The resulting fit between data and model moments, together with the estimated parameters are shown in Table C.6. The results from removing import delays in the thus recalibrated model are shown in the last columns of Table C.5. We find that removing import delays still has a sizeable positive effect on aggregate output, albeit smaller than in our baseline calibration. The reason is that model fit of the benchmark economy is not as good with the truncated moments than it is for the baseline calibration. This has to do with the fact that the calibration procedure of the model has difficulties dealing with derivatives of the moments at the kink at $n = 5$ for all firm types. In the resulting calibration, the fraction and average size of type 3 firms are above their data counterparts, leading to smaller marginal gains from removing the delay friction.[31]

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[31] Another reason for why the aggregate output gains are smaller in the economy without firms that have $n < 5$ is that as the friction is removed, type 1 firms' upgrading behavior shifts a larger mass of firms into a mass point of type 2 firms, where they have less than five employees, thus excluding those firms from the analysis.

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